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Vol. 250 DECEMBER, 1950 No. 6

THE ROLE OF MATHEMATICAL METHODS IN PHYSICAL THEORIES*

BY

EUGENE P. WIGNER[1]

Mr. Chairman, Ladies and Gentlemen:

Permit me first to express my regret at my inability to attend this occasion and to express my great appreciation for the honor which The Franklin Institute has conferred upon me.

About two years ago, Dr. Enrico Fermi spoke to you of the great significance of technological progress for the advancement of science. He pointed out that most of the specialized and often impressively large equipment, which has lately contributed so much to our knowledge of natural phenomena, could not have been created without a vast amount of industrial background and know-how. Similarly, modern experimental physics would be inconceivable without the many small instruments which modern industry mass produces. It is a natural consequence of this latter fact that we use in our laboratories a great deal of equipment, the functioning of which we take for granted but the understanding of which we leave to the manufacturer.

The tools of the theoretical physicist, the various disciplines of mathematics, might appear less spectacular. However, if we understand them properly they present as impressive structures as any made by man. These structures play two roles in the realm of physics which may be called, respectively, formulative and deductive. "The laws of nature are written in the language of mathematics" said Newton and this characterizes the first role mathematical theories play in physics.

* Talk presented at the Medal Day Meeting at The Franklin Institute, October 18, 1950. (Read by Dr. Mary Wheeler Wigner, in the absence of the author.)

[1] Thomas D. Jones Professor of Mathematical Physics, Princeton University, Princeton, N. J.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

Newtonian physics was based only on elementary calculus, but since his time, as physics grew and encompassed an increasingly wide field of phenomena, its laws had to use the language of partial differential equations, of "Ricci calculus," and of operator algebra. This development is by no means at an end, neither does it cease to fill us with the sense of the miracle of the simplicity of nature's laws when expressed in terms of appropriate concepts. As Schrodinger said, this miracle is second only to the greater miracle that man may succeed to understand these laws.

The second role which mathematical methods play in physics is more similar to the role which experimental apparatus plays and to which I have referred earlier. Both tools have been called, jocularly, props for our incompetence. The experimental tools help us by isolating phenomena which our limited intelligence does not permit us to see sufficiently clearly unless they are so isolated, the mathematical tools help us to bridge the gap between the general theory and the phenomenon or group of phenomena in question. Actually, both are more than mere artifices; the general theory would be empty without the two links, mathematical and experimental, between itself and experience. Furthermore, the purposefulness of a well designed experiment, the structure of a well conceived mathematical deduction from the general theory, or of a clever reformulation thereof, inspires in the scientist a sense of pleasure which is second only to the pleasure which he experiences when contemplating a general law of nature.

Gibbs' theory of statistical ensembles, based on the geometry of multidimensional phase space, will probably permanently remain the most elegant and fruitful representative of the deductive role of mathematical theories. However, there are few branches of mathematics which have not found their way into theoretical physics. In particular, mathematical methods which are essentially those of group theory have been applied to problems of crystal physics as early as 1830. I feel proud to have had the privilege of having initiated the application of a very important branch of group theory, that is, that of representation theory, to problems of quantum mechanics.

There is one aspect of the connection between quantum physics and representation theory which has been exploited so far very little and to which I may call attention. This is the point of view which regards the most common orthogonal functions of mathematical physics, such as Bessel functions, Legendre and Jacobi polynomials, as representation coefficients. The Bessel functions occur in the representations of the group of motion in two dimensions, the two dimensional Euclidean group, the Legendre and Jacobi polynomials in the representations of the group of rotations in three dimensional space. The Frobenius-Schur orthogonality relations establish the orthonormal nature of these sets of functions, the representation property their addition theorems. The

possibility of the reduction of direct products of representations provides most of the remainder of the known relations between these functions.

In view of the strong influence which mathematical methods and theories exert upon physics, and conversely, both necessarily and traditionally, one cannot help regretting the aloofness which is growing between mathematicians and physicists. It seems that today we can only rarely convince our mathematician colleagues that we know which problem we want to solve and that our rules of the game are not entirely arbitrary. It certainly is a far cry from the days of close relationship between mathematics and physics under Laplace, Lagrange, the Bernouillis.

It is possible that the growing split between mathematics and physics is unavoidable, that they have grown too big to be understood by one person and that the misunderstandings between mathematicians and physicists are the unavoidable pangs of separation. It is possible, in fact, that mathematics will grow so foreign to us that we shall have to use its results without completely understanding their causes, much as experimental physicists use some of the experimental equipment. It is clear, however, that this should be avoided if at all possible and I feel that we physicists should try to maintain our interest and understanding of mathematics and try to keep the mathematicians interested in our problems. My reason for presuming to speak on this subject is that the closeness of the mathematics and physics departments in Princeton affords me a good opportunity to undertsand the mathematician's point of view when approaching problems of physics.

It would be futile, of course, to propose that physicists use in all their conclusions the rigorous methods of the disciples of the queen of sciences and give their arguments in such detail that the non-physicist may easily follow them. However, we can, and perhaps should, try in our deductive work to (*a*) state our assumptions clearly, (*b*) point to the place at which each assumption enters the argument, and (*c*) state our result and its presumed limitation concisely. In particular, we should avoid arguments which give incorrect results when applied to some other problem but the one to which we apply it, unless it is made evident in the course of the argument why it is not applicable to the second problem. Observance of these rules may, in addition to clarifying our own thinking, contribute to the restoration of a common ground between ourselves and our natural advisors.

Pocket Thermometer.—Convenience is the outstanding feature of the new Thermicator, a pocket-type thermometer introduced by the Tagliabue Instruments Division of the Weston Electrical Instrument Corporation. Design is greatly simplified and improved in this instrument, which can be used either as an armored or as a plain thermometer for temperatures between minus 30° F. and plus 120° F.

The Thermicator consists of an etched-stem glass thermometer, of the finest craftsmanship, and its light-weight, non-corroding protective case. It is slimmer than most fountain pens and is held securely in coat, shirt or vest pocket of any depth by an adjustable clip. Thermicator's metal case, which is open at both ends, has no separable cap to unscrew. Instead, the unique pinch-top of the thermometer projects slightly from the case; the thermometer itself is quickly whisked free or replaced in a single motion. In the case, the thermometer is firmly gripped by strong, untiring spring pressure; it won't come out until deliberately removed.

The Thermicator, complete with metal case, can be suspended from a cord through its eye, and immersed in a test fluid or medium like an armored thermometer. Large apertures in the case help assure better circulation of most light liquids or vapors around the bulb. For general temperature testing, where it is not exposed to the danger of breakage, the thermometer is removed from the case.

Measurements of the Thermicator are: 6 inches long, $\frac{3}{8}$ inch in diameter. Indicating medium: ruby-red spirit. Scale divisions: 2° over instrument range. The entire bore, including expansion chamber, is visible. A flared, pinch-top grip prevents the Thermicator from rolling on flat surfaces.

Medical Diagnosis by Radioactivity.—Radioactivity is now ready for its new role as a major tool of the physician. Although the work with radioisotopes has been largely confined to research and experimental programs in a limited number of institutions, clinical evidence indicates a great number of ways in which the use of radioisotopes will speed accurate diagnosis and provide unequalled opportunities for improved therapy. These techniques will provide maximum benefit as they become generally used throughout the field of medicine.

Among the new diagnostic techniques which show great promise is the localization of tumor tissue, especially in the brain, using radiodiiodo-fluorescein and determining the uptake of this material by means of special detectors. An example is the Isotron, an experimental unit built by Nuclear Instrument and Chemical Corp. and used at Northwestern University Medical School for developing techniques for external counting through the concentration of radioactive dye within the brain and its contiguous structure. Accuracy of 95% in the location of brain tumors seems assured by the results of extensive clinical tests.

GROWING PIEZOELECTRIC CRYSTALS[1]

A. C. WALKER[2]

INTRODUCTION

This paper is a summary of work carried out at the Bell Telephone Laboratories on the growing of large single crystals of three different piezoelectric materials: ammonium dihydrogen phosphate (ADP); ethylenediamine tartrate (EDT); and quartz. Included are illustrations of some basic principles observed in the growing of these crystals, descriptions of improved apparatus for their growth, and pilot plant problems encountered in conjunction with the commercial production of ADP and EDT.

HISTORICAL BACKGROUND

The interest of the Bell System in the growing of piezoelectric crystals dates from the time of World War I. A. M. Nicholson (1),[3] in the Laboratories of the Engineering Department of the Western Electric Co., grew large, single crystals of Rochelle salt, and applied these to such diverse uses as microphones, telephone receivers, frequency controls for electronic oscillators, and modulators of alternating currents (2).

In 1938, Georg Busch (3) and co-workers in Zurich, published the results of an investigation of the dielectric properties of ammonium and potassium dihydrogen phosphates (KDP) and arsenates, four isomorphous crystals possessing piezoelectric properties. Independently the Brush Development Co., in Cleveland, had investigated ADP crystals, and the Bell Telephone Laboratories had investigated KDP crystals. During the recent war the Navy requested this company to grow ADP crystals for submarine detecting apparatus. A like request came to the Western Electric Co., and they set up an installation patterned after the one designed by the Brush Co. The Bell Telephone Laboratories then built a pilot plant of much the same design, to assist the Western Electric Co. in solving problems of crystal growth which naturally developed in this relatively new industry.

After World War II it became apparent that the supply of Brazilian quartz, particularly large-sized crystals, would be inadequate for use in greatly expanding telephone needs. After a search, which had been in progress for some years, it was found that EDT could be used effec-

[1] Based on a lecture given at a Joint Meeting of The Franklin Institute and the Physics Club of Philadelphia, in the Lecture Hall of the Institute, December 7, 1949.

[2] Member of the Technical Staff, Bell Telephone Laboratories, Inc., Murray Hill, N. J.

[3] The boldface numbers in parentheses refer to the references appended to the paper.

tively as a substitute for quartz for these telephone purposes. The experience gained in studying the growth of ADP crystals was invaluable in rapidly getting into commercial production of EDT.

At about the time the Laboratories started work on EDT crystals, information from Germany indicated that quartz crystals had been successfully grown by Prof. R. Nacken (4) at the University of Frankfort. Preliminary experiments made with the method he described, verified his results, but there were serious limitations to the German process. After a year of experimentation, a process was developed quartz which avoided the limitations of the German method. Now large crystals can be grown by a hydrothermal process, and this appears to be useful commercially to supply quartz in case of emergency.

GROWING AMMONIUM DIHYDROGEN PHOSPHATE CRYSTALS

Method

The classical method of preparing a single crystal of a water-soluble salt, as illustrated by ADP, is to saturate a solution of the salt a few

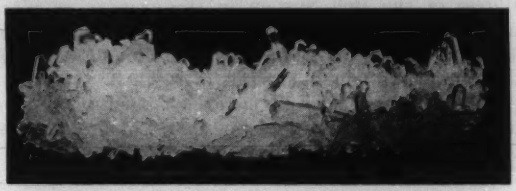

FIG. 1. Mass of spontaneous ADP crystals.

degrees above room temperature and allow it to cool slowly in a quiet place, in a covered crystallizing dish. While still warm, and above the saturation temperature, it is inoculated with a small piece of clear ADP crystal. On cooling below the saturation temperature, the natural faces form on this nucleus, and the crystal grows rapidly on certain faces, less rapidly on others. Within a few days a relatively clear crystal is formed, about $\frac{1}{4}$ in. in cross-section.

Another convenient way is to saturate the salt solution well above room temperature so that when cooled, it has a high degree of supersaturation. Spontaneous nuclei rapidly form and settle to the bottom of the dish. In a few days a large number of interlaced crystals develop, some with quite clear ends. On longer standing, these clear ends attain some length as may be seen in Fig. 1. Small crystals prepared by either of these methods are useful seeds for growing larger crystals.

Apparatus for Growing Large Single Crystals

A commercial method for growing large single crystals of ADP has been described previously (5, 6). The crystals are grown in long, flat

stainless steel tanks filled to about half their depth with salt solution saturated at about 46° C. The seed crystals, in the form of plates, are mounted on rubber covered stainless steel pins fastened to frames located on the bottom of the tanks. The tanks are rocked endwise several times a minute, through a small angle along their long dimension, to promote liquid circulation past the growing faces.

This is termed a dropping temperature process because the system is continuously lowered a fraction of a degree per day. As the growing crystal removes salt from the solution, the loss is balanced by a decrease in salt solubility with lower temperature. The higher the degree of supersaturation, the faster the rate of crystal growth, although there is obviously an upper practical limit to this rate.

The room temperature must be controlled to 0.1° C., at all times, and the air flow must be substantially the same over all tanks. Any variation in air flow over each tank causes a marked spread in temperature between different tanks in a relatively short time. The objective is to grow all crystals in all tanks as fast as possible, without defects. Thus it is important that there be no appreciable spread in temperature between the different units. This puts an unusual requirement on the air conditioning equipment.

ADP crystals were grown commercially in rooms containing as many as 56 tanks per room, arranged in four rows superimposed one above the other in two tiers, one on each side of the room. Such a cell could produce over 2200 crystals in a 40 day cycle, each crystal being about 9 in. in length by $1\frac{5}{8}$ in. in cross-section.

Growing Solution

Solubility.—It is possible to prepare a solution of ADP, saturated at a given temperature, by mixing the salt with water in proportions determined from published solubility data. The data reported by Buchanan and Winner (7) may be summarized by the following equation:

$$S_T = 180 + 4.55\ T,$$

where S_T is the solubility of ADP at T° C., in grams per 1000 grams of saturated solution. Unpublished data from the Naval Research Laboratory give this equation as:

$$S_T = 171 + 4.70T$$

between 20° and 50° C.

Preparation of Saturated ADP Solution.—It is desirable to prepare the solution saturated at a given temperature by first dissolving somewhat less than the required amount of salt in water and then vigorously stirring the mixture in the presence of an excess of finely divided salt,

at the desired temperature. Thus equilibrium is approached from the unsaturated side. When the specific gravity of this solution remains constant after three successive measurements, taken at hourly intervals, this is satisfactory evidence that the solution is saturated at the equilibrium temperature.

TABLE I.—*Physical Properties of Ammonium Phosphate Solutions.**

At Saturation Temperatures of 41°, 46° and 50° C.

Salt or Acid-Salt Ratios (in per cent)	Specific Gravity Sat. Solutions at T° C. by Hydrometer	Measurements on Solutions Diluted 1:1 with Water at 26° C. pH	Specific Gravities by Hydrometer
$\frac{(NH_4)_2H\,PO_4}{(NH_4)\,H_2PO_4} \times 100$			
	T = 41° C.		
0.0	1.2073	3.70	1.1113
2.1	1.2098	3.99	1.1128
6.2	1.2155	4.35	1.1165
10.2	1.2231	4.58	1.1210
11.0	1.2250	4.65	1.1222
$(H_3PO_4)/(NH_4)H_2PO_4$			
0.75	1.2081	3.62	1.1119
1.52	1.2090	3.42	1.1124
	T = 46° C.		
(Sec./Pri.) × 100			
0.0	1.2212	3.69	1.1197
1.95	1.2228	3.93	1.1210
5.9	1.2278	4.29	1.1240
	T = 50° C.		
0.0	1.2322	3.66	1.1268
0.3	1.2324	3.68	1.1272
1.0	1.2329	3.77	1.1275
2.1	1.2337	3.90	1.1278
3.3	1.2350	4.02	1.1287
4.4	1.2361	4.11	1.1294
6.6	1.2395	4.29	1.1311
8.9	1.2440	4.45	1.1341
10.7	1.2480	4.54	1.1367
$(H_3PO_4)/(NH_4)H_2PO_4$			
1.3	1.2335	3.45	1.1275
2.3	1.2347	3.30	1.1280
3.3	1.2358	3.14	1.1289

* Specific gravities were measured with a 60°/60° hydrometer. The readings are good to ±.0003.

Table I contains specific gravity data of saturated ADP solutions at 41°, 46° and 50° C. These are simply guides in the rough preparation of the solution, to be followed by stirring with an excess of salt to insure equilibrium.

Figure 2 is a plot of these data. One important fact to be derived

from this plot is that a solution saturated with ADP has a minimum specific gravity when the ratio of ammonia to phosphoric acid is that of the ammonium dihydrogen phosphate. The addition of an excess of either the acid or the base increases the solubility of the salt, and of course changes both the specific gravity and the pH. Therefore both pH and hydrometer readings must be taken to determine the saturation temperature of an ADP solution.

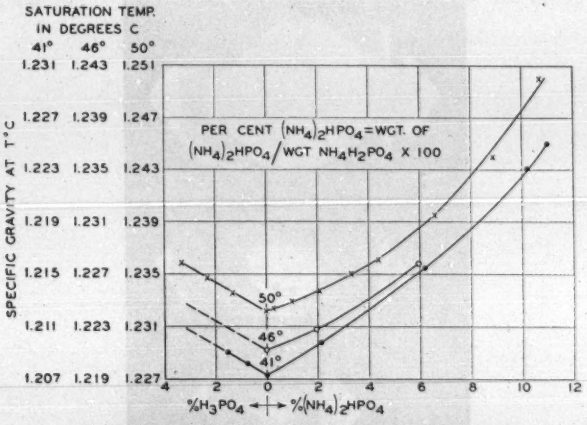

FIG. 2. Effect of pH and temperature on the solubility of ADP.

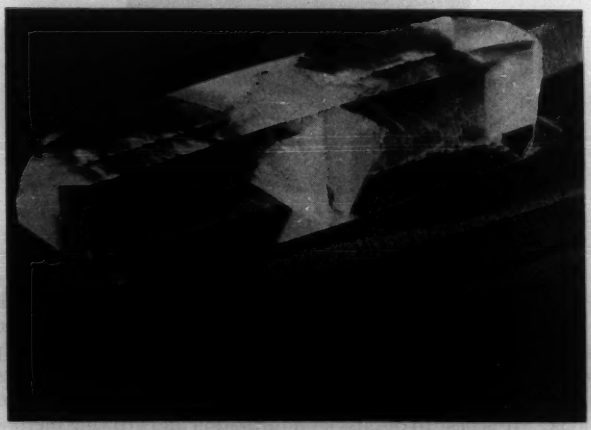

FIG. 3. Commercial bar crystal of ADP.

Crystal Seed Formation

Any piece of single crystal is a possible seed for a new crystal bar, regardless of the character or complexity of its bounding surfaces. However, clear growth takes place only on certain natural faces, and so the best seeds are those having well-developed faces. The atoms of which crystals are composed differ in a variety of ways. Some of the important differences which may affect crystal growth are interionic

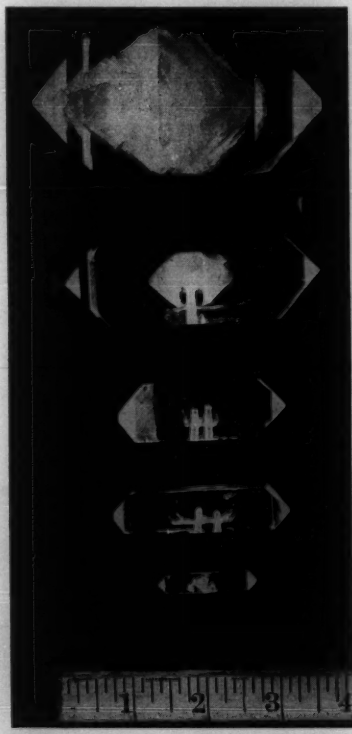

FIG. 4. Sequence of ADP crystals showing cross section increase.

and intramolecular forces, and the bond-energies normal to the growing faces. Therefore, each kind of crystal may have individual growth characteristics. There are also many modifying effects caused by impurities, temperature, and rate of crystallization. While there are some general rules which apply to many crystals, there are also numerous exceptions. It is best to discuss seed formation and growing methods of each of these three crystals separately.

Seed Enlargement

To get a commercially useful ADP bar of about 1½ in. in thickness (see Fig. 3) requires a seed of these dimensions. To achieve this from the small seeds requires a number of growing cycles. The saturated solution of pure ADP salt has a pH of 3.66 to 3.70, see Table I. In this solution, growth takes place only on the end pyramid faces of the crystal. A seed such as that shown in Fig. 1 forms a slightly longer, acicular crystal like the bottom one in Fig. 4. If the pH of the saturated solu-

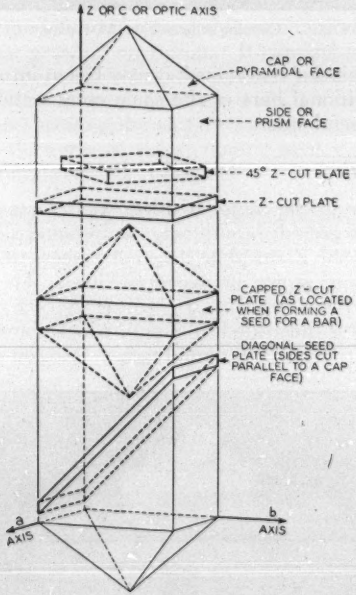
Z OR C OR OPTIC AXIS
CAP OR PYRAMIDAL FACE
SIDE OR PRISM FACE
45° Z-CUT PLATE
Z-CUT PLATE
CAPPED Z-CUT PLATE (AS LOCATED WHEN FORMING A SEED FOR A BAR)
DIAGONAL SEED PLATE (SIDES CUT PARALLEL TO A CAP FACE)
a AXIS
b AXIS

FIG. 5. Sketch of ADP crystal bar showing several kinds of cut sections.

tion is increased to about 5, by addition of either ammonia or the secondary ammonium phosphate, growth may be promoted on the prism faces. The second, third and fourth crystals from the bottom in Fig. 4 show this enlargement. Uusally such growth is accompanied by some increase in length. Unfortunately, a solution of pH 5 is more unstable as regards spontaneous seeding than one of pH 3.7. After a week or so of growth in the pH 5 solution it is necessary to remove the somewhat enlarged bar and repeat the process. Six months of repeated planting and harvesting may be required to enlarge the original ADP seed to the size shown in Fig. 3.

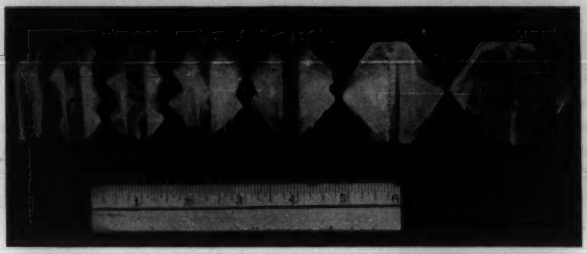

FIG. 6. Capping sequence for ADP plates.

Once a bar of this size is obtained, it may be cut into plates for use as seeds to grow additional bars of the same cross-section. Figure 5 illustrates certain useful cuts.

Growing Considerations

The *Z*-cut plate in the center of Fig. 5 represents an economically useful seed. Clear growth occurs only on pyramid faces of the ADP bar, and obviously the *Z*-cut plate does not possess these faces. ADP crystals are therefore grown in two operations. The first is termed capping, the second bar growth.

Capping.—This is illustrated in Fig. 6, which shows the successive stages in the formation of the pyramid faces. Motion of the solution is

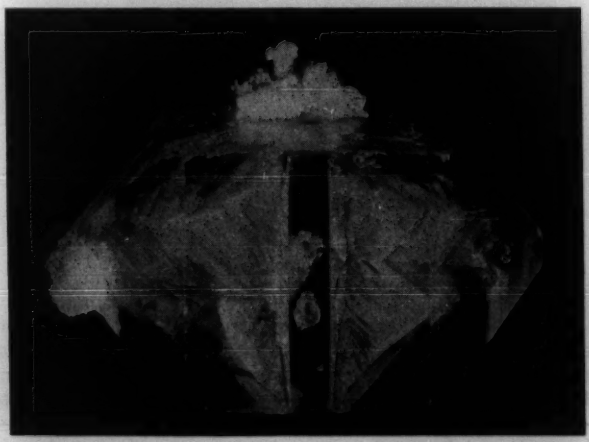

FIG. 7. Leakage of entrapped mother liquor leaves mass of misoriented crystals on surface of capped seed plate.

across the major Z-cut (*i.e.* {001}) faces, and this induces a maximum of circulation on the edges and corners of these faces. Growth of the pyramids is very rapid, starting from tent-like corners. These enlarge until they choke off circulation within the middle area of the plate faces. Finally these surfaces converge to thin-walled pyramids covering each Z-cut face. Capping is completed on $1\frac{1}{2}$ in. square plates in about 28 hr., under optimum conditions. Enclosed within these pyramids are appreciable amounts of salt solution and misoriented deposits from the occluded mother liquor, hence the cloudy appearance of the capped seed plate.

After capping, the units are removed from solution, wiped dry and prepared for the bar growing operation. If capping is stopped before the pyramid walls are completed, the trapped mother liquor will effloresce through cracks or openings in the walls, giving clusters of misoriented crystals and spoiling the surface for subsequent growth. See Fig. 7. The wall thickness of the pyramids need not be great, it is sufficient to have simple surface continuity.

FIG. 8. Effect of temperature on capping of ADP plates.

It is helpful to have certain impurities dissolved in the solution. Iron[4] in concentrations of the order of 0.002 M., stabilizes the solution against spurious seeding, limits lateral growth, and promotes more rapid capping. Aluminum sulfate, to the extent of 0.005 M. Al concentration, makes for even more rapid capping.

The temperature influence on capping is somewhat critical. If the operation is started at 50° C., with the dropping temperature procedure, capping is slow and inefficient, taking up to a week to complete. Even then, plates with misoriented center sections may fail to complete the pyramids. Starting at 41° C., capping is remarkably rapid and effective, even with poor plates. See Fig. 8. The capped seed plate on the left was started at 41° C., the one on the right, at 50° C. The longer the prism faces, the slower the capping operation.

[4] Kjellgren and Jaffe, U. S. Patent 2,452,576 (1948).

Bar Growth

Capped seed plates are carefully selected for bar growth to avoid incomplete or badly formed pyramid ends. These are planted in the solution saturated at about 46°–50° C., so that circulation of the liquid will be against the pyramid faces. This bar growing solution must be free from ions such as Fe, Cr, Al, Ba, Sn, SO_4, to name only a few of the more undesirable ones. Clear growth on the pyramid faces takes place at rates determined primarily by the degree of supersaturation maintained, but to some extent by the amount and uniformity of circulation of the solution.

The cap on the left in Fig. 8 is about $2\frac{1}{2}$ in. in length, measured along the *Z*-axis. It can be grown into a bar 9 in. long in 40 days. The growth rate on each end is 0.08 in. per day. Of the total of $3\frac{1}{4}$ in. of

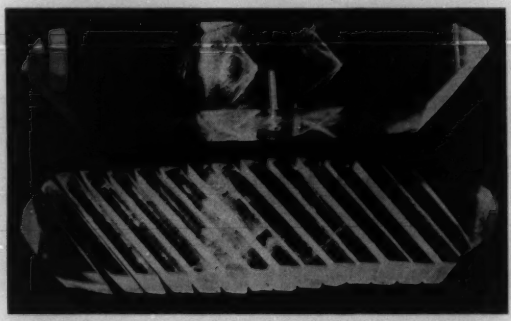

FIG. 9. ADP bar cut into diagonal plates—cut parallel to cap face.

clear growth on each end, only 2 in. is useful for piezoelectric plates. The pyramid ends are waste, not being economical for use as seed material in capping. The center, cloudy sections are cut into plates for another capping operation.

Diagonal Seed Plates

Only seeds all of whose faces have the orientations of natural faces can be used to grow clear bars, because clear growth takes place only on faces with these orientations. This important principle led to a means of growing clear, transparent bars with no cloudy double pyramid. Figure 9 shows a bar cut into diagonal plates, the angle of cut being parallel to a natural pyramid face. This angle is 44°50′ with respect to the optic or *Z*-axis of the ADP crystal. This cut is also illustrated in Fig. 5. Such diagonal plates have all natural faces, or ones parallel to natural faces. Clear growth takes place at once on such faces, without a preliminary capping operation. Figure 10 shows a bar grown from a

diagonal seed plate. The faint outlines of the original plate edges are caused by slight solution of these edges during the initial washing period after planting, while the solution is still above the saturation temperature.

FIG. 10. ADP bar grown from correctly cut diagonal plate.

This washing dissolves misoriented particles and rounds the edges of the plate. Subsequent growth is more rapid at these rounded edges, until they are again filled out, and this rapid growth includes some liquid. The resulting bar has clear useful material from end to end, for the slight cloudiness of the edges of the plate are on or near the outer surfaces.

Figure 11 shows a bar grown from a diagonal plate cut 1° off the 45°50′ angle of the pyramid face. The incorrect angle fills rapidly with cloudy growth before clear deposition occurs. This emphasizes the

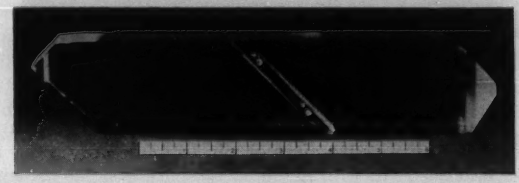

FIG. 11. ADP bar grown from diagonal plate cut 1° off the correct diagonal angle.

need for accuracy in cutting the diagonal plate to the proper angle. Also the larger the area of a growing surface the more care must be taken to control temperature during the initial hours of growth, and the more margin must be left in operating below the maximum growth rate to avoid crystal defects. These limitations offset the apparent advantage of using diagonal seed plates in growing ADP.

ADP Crystal Defects

Crystal defects are legion. The crystallographer or geologist is usually much more interested in an imperfect crystal than in one without flaws. The crystal grower, in his attempts to grow large single crystals, particularly for electrical purposes where even small defects are serious, views flaws with a jaundiced eye. Three common defects of crystals are: veils, tapering and misoriented growth. All three are shown by the top crystal in Fig. 12.

Veils.—Cloudy lines or areas in an otherwise clear portion of a crystal are usually termed veils. These are due to excessively high growth rates, or to circulation defects. The cloudiness comes from occlusion of mother liquor when some points on the growing surface enlarge more rapidly than others, creating discontinuities in the smooth deposition of materials. Correction of the condition causing veiled growth usually results in clear deposition beyond the defective area.

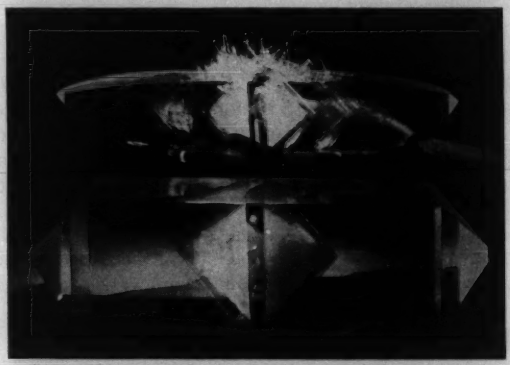

FIG. 12. Good and defective ADP crystals.

Tapering.—Adsorption of impurities from the solution may cause a reduction in the area of the growing surfaces, leading to a tapered crystal. Dissolved iron, chromium or tin, to name only a few undesirables, cause marked reduction in the cross-section of the growing faces of the ADP crystal. For example, as little as 1.8×10^{-3} molar concentration of Cr^{+++} causes a 16° taper (8). Not much more than this amount of Cr was present in the solution in which was formed the top crystal in Fig. 12. Much less than this amount, if any, was present in the solution producing the bottom crystal in this picture. This bottom crystal is one of the most perfect, large ADP bars ever grown. It is 3 in. in cross-section by 12 in. long and weighs over 6 lb. Together with 20 others equally perfect it was grown in 72 days in a pilot plant unit.

In Fig. 13, growth was nearly completely inhibited on the ADP

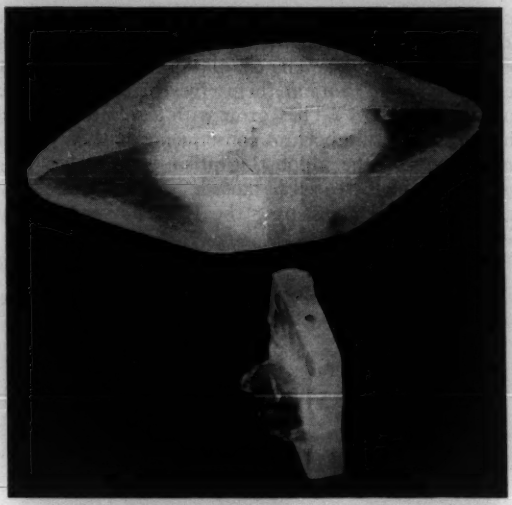

FIG. 13. Effects of chromium and tin in growing solution on the growth of ADP crystals.

plate in front, by the presence of only 0.005 *M*. $CrCl_3$. The other crystal in Fig. 13 is strongly tapered due to the presence of Sn. The tapered surfaces have a roughened appearance, which seems to be characteristic of this impurity.

Misoriented Growth.—At high supersaturations, ADP solutions become unstable and multitudes of long, needle-like crystals make their appearance. These settle on the top faces of the growing bars. A severe "snow-storm" of such nuclei caused the misorientation on the top crystal in Fig. 12.

Such a severe case of misorientation can usually be avoided by promptly raising the temperature of the solution above the saturation point to dissolve most of the nuclei. Their complete removal is very difficult without some solution of the bar edges, and this leads to the

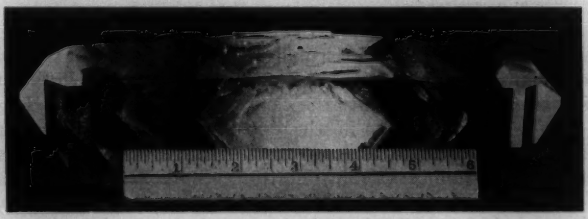

FIG. 14. Splinter growth on ADP bar.

veil defect. When only an occasional spontaneous seed appears it usually adheres to the top face of the bar, although nuclei sometimes adhere to other prism faces. The circulation conditions are such in the crystallizer that the nuclei orient themselves with their long axis nearly parallel to that of the bar. They grow with a splinter-like appearance, like that in Fig. 14. An important consequence of their presence is that they induce growth on the supporting prism face. This is one way in which crystals may be increased in cross-section. It is the only effective way thus far found for thickening the EDT crystal—see section on Crystal Growing Conditions.

FIG. 15. Reciprocating rotary crystallizer with EDT crystals.

Misoriented splinters do not usually extend to the end of the growing bar, but their influence is shown by cracks which originate in the area of misorientation, and spread into the clear end. Sometimes these cracks occur during growth, even in the constant temperature apparatus. They nearly always occur on harvesting. It is believed that this misoriented growth induces strains in the crystal. In some cases the strains are severe enough to cause cracks without appreciable temperature change. If a marked change occurs, as in harvesting, the strain is

aggravated by unequal coefficients of expansion in different directions through the composite crystal and cracking is almost certain to occur. Thus it is difficult to get enlarged crystals with clear, uncracked ends as seeds during the initial program of building up material for commercial use.

Chromium can be detected in an ADP crystal with splinter-growth, even if present in very small amounts. This ion is adsorbed in the misoriented lattice on the prism face. If viewed edgewise, the misoriented layer has a greenish tint.

New Apparatus for Growing Crystals

The experience gained in the work on ADP led to the development of two new pieces of crystal growing apparatus. One of these is the

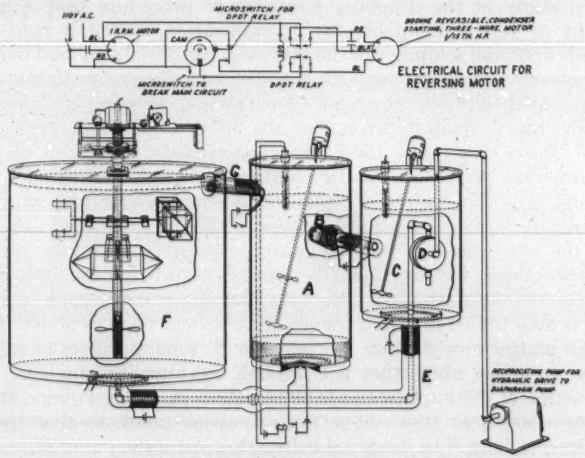

FIG. 16. Sketch of constant temperature crystallizer.

rotary reciprocating crystallizer shown in Fig. 15. The other apparatus is a constant temperature adaptation of this. Figure 16 is a sketch of the pilot plant model.

Rotary Reciprocating Crystallizer.—This apparatus was developed by Mr. A. N. Holden (9). Crystal seeds are affixed to a spider rotating through the solution, actuated by a motor drive mounted on a stainless steel lid. The crystals rotate at a speed of about 20 rpm., for a short interval in one direction, then for an equal interval in the opposite direction. The solution begins to flow in the direction of rotation, because the spider and crystals act as a propeller. Reversal of spider direction is followed by a slower reversal of solution flow. It takes 30

seconds for the solution to reach a flow rate nearly equal to the speed of the spider and this determines the time interval between reversals.

Spurious seeds form readily at the surface of a supersaturated solution, particularly if the surface is in motion. This limitation of the rocking tank method is avoided with the rotary crystallizer. Condensation of water vapor on the under side of the cold lid is followed by return of some water to the solution surface. Thus the quiet, relatively unstirred surface layer remains slightly unsaturated. Unwanted seeds will not grow in this narrow zone. If seeds do form at the surface due to control failure, most of them settle to the bottom, but a few may adhere to the top faces of the bars. Those which reach the bottom redissolve, because there is a thin layer of unsaturated solution in contact with the heated bottom.

Constant Temperature Method of Growing Crystals.—It was clear from a study of the dropping temperature procedure that a limited amount of salt is available for crystallization during a run. This depends upon the solubility of the salt used, the initial and final temperatures of crystallization, and the volume of solution available. This limitation led to the development of a constant temperature apparatus, utilizing the desirable features of the rotary reciprocal crystallizer. Figure 16 is a sketch of a 150-gallon capacity pilot plant unit on which the first tests were made of the constant temperature procedure (10).

Tank *A* is the saturator. Solution is stirred constantly at a fixed temperature in contact with an excess of the finely divided salt. Because the rate or flow in the system is only 10–15 gal. per hour, the solution is practically saturated in *A* before it flows through the filter *B*, and the heated insulated pipe line, into the superheater tank *C*. The liquid is maintained in *C* at a temperature several degrees above that in *A*. No matter how fine are the pores in *B*, some particles of salt will dissolve to a size where they get through the filter into tank *C*. It is the function of this superheater tank to maintain a large volume of solution for a sufficient time above the saturation point, so that the fine particles of solid will be dissolved before they enter the pump *D*.

The solution is delivered by this pump through the heated insulated pipe line *E*, into the crystallizer tank *F*. This tank is controlled at a temperature about 1° C., below that in *A*, and this temperature difference determines the degree of supersaturation in which the crystals grow. Radiation of heat from the bare, stainless steel walls of *F*, cools the entering liquid rapidly enough so that it mixes with the solution in *F* without raising the tank temperature. Partially depleted solution overflows by gravity through the line connecting *F* and *A*. Different solution levels are maintained in *A*, *C* and *F*, by the pump, the slight resistance of filter *B*, and the position of the overflow in F.

Both of these new types of crystal-growing apparatus were used first to grow ADP crystals. The first experiment in the constant

temperature apparatus was the growing of four 43-lb. ADP bars in a period of four months. One of these crystals is shown in Fig. 17. It is slightly over 6 in. in cross section and 22 in. long. The absence of light in most of the end sections beyond the seed cap indicates the perfection of growth. Some splinter growth formed on the top faces due to a mechanical failure in the early stages of operating this new equipment. The misoriented growth is visible in the picture as a light area along the bottom. The bottom face in the picture was the top in the growing operation.

This experiment demonstrated the practical utility of the constant temperature design and was an important factor in a later decision to use this type of equipment for growing EDT crystals commercially.

GROWING EDT CRYSTALS

Selection

Out of an investigation [5] to find a substitute for quartz for telephone purposes, two possibilities were suggested: ethylenediamine tartrate

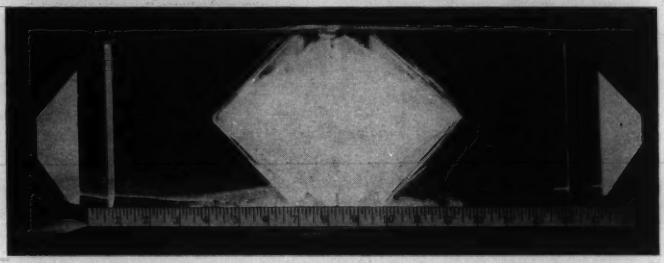

FIG. 17. Large ADP bar—6 in. in cross section by 22 in. long, weight 43 lb.—grown in 4 months.

(EDT) and dipotassium tartrate-hemihydrate (DKT). Certain undesirable physical properties of the DKT crystal, as well as difficulties encountered in growing large single crystals of this salt, led to the decision to concentrate on the growing of EDT.

Methods of Growing EDT Crystals

Dropping Temperature Procedure.—The rocking tank method employing the dropping temperature procedure was considered unsuitable for the EDT crystal. The supersaturated solution appeared to have a much greater tendency to crystallize spontaneously than did ADP, at the moving surface of the solution. This difficulty was readily overcome in the rotary reciprocal crystallizer, using the dropping temperature procedure. Figure 15 shows this apparatus in use growing 6 EDT crystals—12 can be accommodated equally well. EDT crystals of the

[5] By A. N. Holden and W. P. Mason, Bell Telephone Laboratories, Inc.

size and clarity shown in Fig. 18 are readily obtained with this apparatus. Unlike the ADP crystal which grows in both directions along the long, or *Z*-axis, the EDT crystal grows appreciably only along one direction, only in the negative direction along the *b* (or *Y*) axis.

Constant Temperature Procedure.—The results obtained in growing large ADP crystals in the constant temperature apparatus showed the value of this equipment for the commercial production of EDT. Ac-

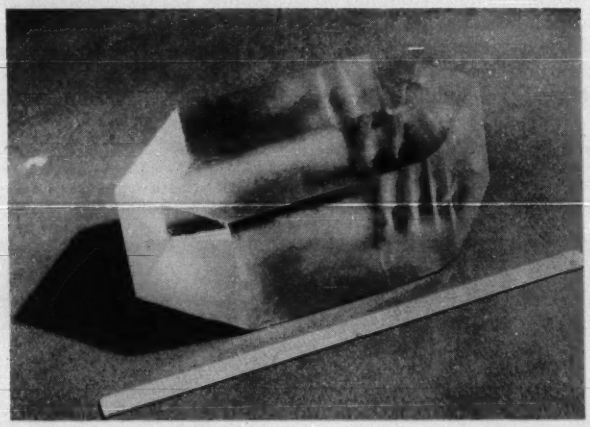

FIG. 18. Clear EDT bar.

cordingly a 44-tank unit was built by the Western Electric Co., at Allentown, Pa., each tank having a capacity of 200 EDT crystals. This crystal grows at a rate of about $\frac{1}{32}$ in. per day in the one direction, and a growing cycle of 3 months is economically useful.

EDT Growing Solutions

Solubility of EDT in Water.—No published information has been found on the solubility of EDT in water. During the first two years of this investigation the only data available were those contained in

TABLE II.—*Solubility of Ethylenediamine Tartrate Salt in Aqueous Solution at pH = 7.15.*

Temperature	% EDT by Weight	Specific Gravity
T° C.		*T*° C.
25.3	57.3	1.2897
30.1	58.8	1.2925
31.1	58.7	1.2925
32.1	58.9	1.2932
33.2	59.2	1.2938
40.2	60.9	1.2972
50.0	63.1	1.3042

These data were obtained on one solution, successively saturated at increasing temperatures.

Table II. By inspection, these data show that EDT is extremely soluble in water and has a rather small change of solubility with temperature.

For reasons which will be explained later under pH control, these data were obtained with an EDT solution having a pH of 7.15, although the "neutral" solution of EDT, made up from pure salt has a pH of 6.0 when saturated at 50° C.

Preparation of Saturated EDT Solution.—Either of two methods may be used to prepare an EDT solution. The simplest is to add solid tartaric acid to a water solution of ethylenediamine. Because considerable heat is evolved as the acid and base react to form the salt, the mixture should be vigorously stirred during the addition. The acid should be added at such a rate that the temperature of the liquid will not exceed 60° C. The second method is to add pure, recrystallized EDT monohydrate to water in the proper proportions.

Acid-Base Mixture.—Ethylenediamine has properties much like ammonia, and is available commercially as a liquid of constant boiling composition, containing about 68–70 per cent by weight of the base in water. The amounts of base and acid used should be somewhat in excess of that required to saturate the solution at the desired temperature. When mixing is complete the solution is cooled to the saturation temperature and maintained at this point with stirring so that the excess EDT will crystallize out. After 24 hr. the solution should be saturated.

Tartaric acid is optically active, the two forms of which were discovered by Pasteur in his historical investigations on crystal asymmetry. Only the dextro, or right hand form of the acid is readily available. It is obtained as a by-product in grape fermentation, and is produced commercially in great quantities from this source. The granular form is preferred to the powder form for making the EDT solution as it reacts without so much of a tendency to lump. The levo, or left hand form of the acid can be prepared, but only by a difficult and expensive process. This is perhaps fortunate, for if the levo form were also readily available, mixing of the two kinds might occur, with unfortunate results.

Solution from Re-crystallized EDT Monohydrate.—This form crystallizes readily from solution at room temperature. The mother liquor is easily removed either by filtering or by centrifuging, and the resulting crystals are very pure. They dissolve to give a water-white solution. The anhydrous form can only be recrystallized above 41° C., once the hydrate has made its appearance in the operations. The anhydrate crystallizes with occlusion of mother liquor to such an extent that the solution of recrystallized salt is always more or less colored.

The hydrate may be used to make up pure EDT solutions for solubility studies, and may be used for short periods of time without the interfering effects of decomposition products.

Decomposition of EDT Solutions.—Heat and sunlight both cause

brownish decomposition products of EDT. In solution these colored decomposition products may be removed by filtering with charcoal. Activated charcoal, and diatomaceous earth, a useful filtering aid, introduce no undesirable impurities into the solution.

Prepared from an acid-base reaction, the solution is always colored, but is suitable for growing crystals without decolorizing. There are advantages, however, in removing the color. Commercially it is necessary to follow the growth of crystals visually through a glass port in the lid of the crystallizer. Too much color obscures the process. Also as the age of the solution increases, so also does the color, and in some cases this is accompanied by a decrease in growth rate.

While it is known that some form of decomposition seriously inhibits growth rate, little is known about the materials responsible. If the solution which shows a drop in growth rate with time is decolorized, the growth rate is likely to improve somewhat, but not to the initial value.

Decomposition occurs rather rapidly at crystal growing temperatures which are about 41° C. to avoid hydrate formation, but the rate seems to drop off with time. This decomposition causes a progressive rise in the specific gravity of the solution at a given saturation temperature. For example, each time a used solution is resaturated at 50° C., for another run, the mixture in equilibrium with solid EDT shows an increase in specific gravity. See Table III.

TABLE III.—*Increase in Specific Gravity of Saturated EDT Solution.*

Date of Saturation	Specific Gravity at 50° C. (solution saturated with EDT)
2/18/49 (New solution)	1.300
4/28/49 (65 days use)	1.309
8/28/49 (165 days use—total)	1.312
2/15/50 (362 days use—total)	1.314

Resin Formation in EDT Solutions

Changes, such as indicated by these data, are believed to be largely chemical condensation. Decomposition products, both clear and colored, are termed "resin" for want of a better name. A superficial examination of this resin indicated it to be largely a monomeric condensation product of ethylenediamine tartrate. Whatever its composition, it is extremely soluble in water.

A solution of the resin in water, relatively free from EDT salt, is obtained by precipitating most of the EDT as bitartrate, from a solution made acid to a pH of 3.2 with tartaric acid. Such a resin solution, when added in specific amounts to portions of a fresh, colorless EDT solution, gave specific gravities as shown in Table IV, when each solution was saturated at 50° C., at a pH of 7.2.

TABLE IV.—*Effect of Resin Content on Specific Gravity and Growth Rate of EDT Solutions.*

Resin Content		Rate of Growth, Inches per Day	No. of Days Run	Specific Gravity—50° C. Solution Saturated with EDT
Estimated*	Analyzed*			
3.0	2.4	0.038	65	1.3035
6.0	6.8	0.041	64	1.3070
9.0	8.2	0.042	66	1.3100
14.0	14.8	0.039	68	1.3175

* The estimated values were based on the amounts of added resin. Analyzed values were determined by a method to be described.

These data show that the relation between added resin and increase in specific gravity is substantially linear for an EDT solution saturated at 50° C. In this case the resin did not reduce the rate of growth of the crystals. However, if EDT bars are grown in a solution with a pH of 8, made up by adding an excess of ethylenediamine, some form of decomposition builds up which does reduce growth rate appreciably. This leads to the not unexpected conclusion that several kinds of decomposi-

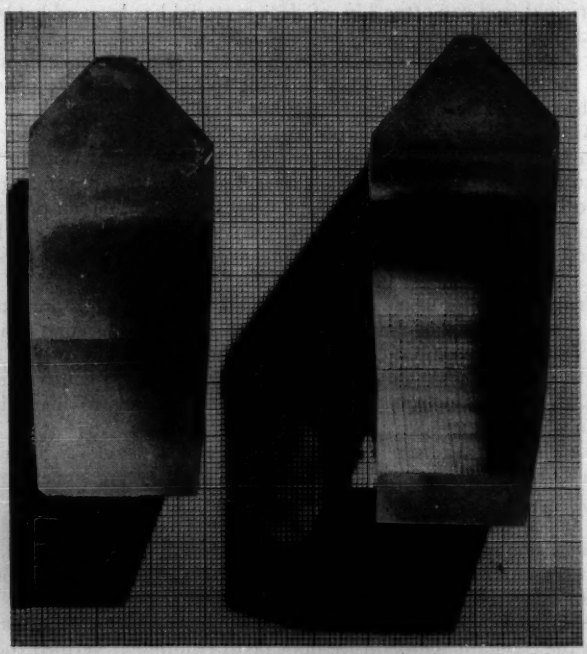

FIG. 19. Taper effect—EDT crystals grown at different pH values. At right: pH = 5.9; at left: pH = 6.9.

tion products form in the EDT growing solution, one or more of which have growth inhibiting properties. To evaluate this property of EDT solutions requires more time and effort than were available on this project.

Specific gravity measurements in Table II may be used to judge if an EDT solution is saturated at a given temperature, *only* if the solution has been made up fresh from pure recrystallized EDT monohydrate, and is practically colorless. Solutions kept for some time above 35° C., or made up by the acid-base reaction, contain indeterminate amounts of resin. This so adds to the specific gravity of the saturated solution that the true salt content cannot be judged by hydrometer.

pH Control of Solution.—The pH of a saturated solution of pure EDT salt is about 6.0, measured on a 1:1 diluted sample at 25° C. At this pH the crystal grows with as much as 20 per cent taper on the {001} faces—see the right hand crystal in Fig. 19. This percentage is an arbitrary figure based on measurements of the crystal across these faces, thus the thickness is 20 per cent less at the tip than at the base of the original seed cap after 3 in. of clear material has been added beyond the end of the seed.

It was found that the addition of an excess of ethylenediamine to bring the pH to 7.2 reduced the tapering to between 2–5 per cent. At 7.5, the taper is less than 2 per cent, but is still visible.

Misoriented or splinter-type growth, like that described for ADP crystals, may occur on either the {001} or {100} faces of the EDT crystal and will induce growth on these faces. Such abnormal growth on the {001} faces occurs on enough crystals in commercial production to more or less offset the undesirable taper at a pH of 7.2. A better solution to the taper problem was found using ammonia as a pH control agent.

Use of Ammonia as a pH Control.—Old EDT solutions are likely to have a strong odor of ammonia. It was logical to assume that this base could be a decomposition product of the system, and this led to the thought that ammonia might be a suitable base for pH control, in that it might reduce the rate of resin formation leading to liberation of ammonia.

Experiments were made with pH values of 7.2 and 7.5, using ammonia as the excess base material, and the results were somewhat surprising. At a pH of 7.5 there was no evidence of tapering on the {001} faces, and at either 7.2 or 7.5 the crystals grew more perfectly and freer from misorientation than when excess ethylenediamine was used. An even more important result was that the growth rate could be doubled with an excess of ammonia present.

Repeated experiments in both the dropping temperature apparatus and in the constant temperature pilot plant equipment verified the fact that rates as high as 0.06 in. per day could be obtained with no danger of spurious seeding.

Definite information is lacking on the effect of ammonia on resin formation, but it was found that the constant temperature pilot plant apparatus could be used to advantage to keep this under some control. As the solution aged and became darker in color, the growth rate also diminished appreciably. This rate could be restored to a value only a little less than the initial one, in the presence of ammonia as the pH control agent, by a continuous 24-hr. filtering cycle with charcoal and filter aid at monthly intervals. The filtration is carried out in the superheater tank without disturbing the continuous operation of the unit.

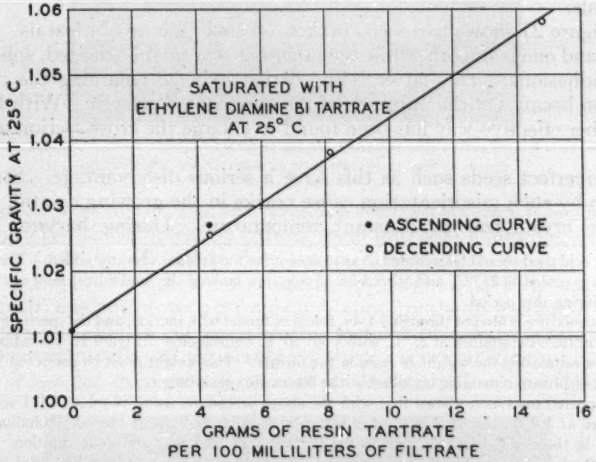

FIG. 20. Calibration curve for resin analysis in EDT solution.

Resin Determination.—In practice, the inhibiting effect of some resin constituents on growth is controlled by replacing the solution with a fresh one at 6 to 9 months intervals. A fair proportion of the salt in the discarded solution can be recovered by crystallizing it out as the monohydrate, which is readily freed from mother liquor by centrifuging. A simple and rapid method has been worked out for estimating the resin, to help decide when to replace a solution. This method does not attempt to determine the composition of the resin, and it may require modification as more knowledge is obtained as to the character of this decomposition product.[6]

[6] The method is as follows: Dilute a 50 ml. sample of the saturated EDT solution with an equal volume of distilled water. The acid salt of EDT which forms when the pH of the solution is brought to 3.2, is soluble to the extent of about 20–30 grams per liter at room temperature, as compared with a solubility of over 500 grams per liter of the neutral salt. Thus most of the EDT may be precipitated as bitartrate. This is done by adding an excess of tartaric acid of 1.26 specific gravity to the diluted sample, until the pH is slightly above 3.2. The volume of

Seed Formation

An EDT solution saturated at 50° C., containing about 63 per cent salt, with a pH = 7.2, is suitable for preparing seeds. If such a solution is allowed to cool slowly from about 52° C., spontaneous seeds form, somewhat more slowly than in ADP solutions because of the high viscosity of the solution, about 240 millipoises. After a period of about a week, a large mass of crystalline material will be found in the bottom of the jar. Some of the single clear needles extending out of the mass will be well formed and may be broken off for use in growing larger crystals.

Figure 21 shows two seeds broken off such a mass of crystals. The left hand one is smooth, while the other has several misoriented, splinter-like adhesions. This latter is the better seed for enlarging the cross-section because of the influence of the misoriented growth. With EDT no other effective way has been found to enlarge the cross-section of the bar.

Imperfect seeds such as this have a serious disadvantage. Strains set up by such misorientation cause cracks in the growing crystal, even in the crystallizer at constant temperature. During harvest their

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tartaric acid used should be recorded so as to determine the amount of water added. The thick mixture is cooled to 25° C., and stirred for 15 minutes, making the final adjustment of the pH to 3.2 during this period.

The mixture is filtered through a 4.-in. Buchner funnel with suction, and the specific gravity of the filtrate is determined at 25° C, with a 60°/60° F. hydrometer. Figure 20 is a calibration curve for estimating the weight of resin in the filtrate. This weight must be corrected by the amount of filtrate remaining occluded in the bitartrate precipitate.

Repeated tests have shown that with the above procedure, using 50 ml. of EDT solution saturated at 50° C., the total amount of water in the mixture is about 110 ml., including that present in the original sample, that in the tartaric acid, and that added for dilution. Only about 90 ml. is recovered in the filtration and the weight of resin found from Fig. 20 should be multiplied by the factor 110/90. With different volumes of solution and reagents the correction factor will have to be redetermined.

To find the weight of resin as it was present in the growing solution at a pH above 7 it is necessary to correct the observed figure by the amount of tartaric acid combined with the resin at pH = 3.2. Assuming the molecular weight of the acid salt of the monomer resin as 325, and the free monomer as 192 the resin is obtained by the following equations:

$$\% \text{ Resin} = \frac{\text{Weight of resin as monomer} \times 100}{50 \times \text{sp. gr. of EDT sample}}.$$

$$\text{Weight of Resin Monomer} = \text{grams resin as tartrate} \times \frac{110 \times 192}{90 \times 325}.$$

This method was checked with a modified van Slyke nitrogen determination, with the following results:

Resin Content	
van Slyke Method	Specific Gravity Method
2.0%	1.86%
12.0%	11.45%

At resin contents above 8%, the precipitation of the bitartrate EDT salt is retarded somewhat by resin, and the method is modified by using more dilution water.

tendency to crack is so great, due to the large temperature coefficient of the EDT crystal, that only a few bars can be removed successfully without these defects.

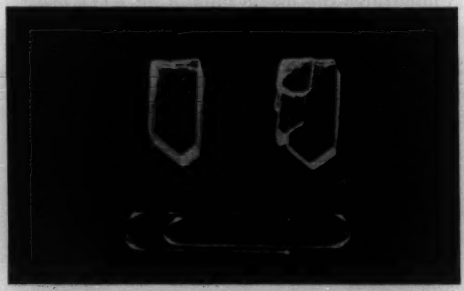

FIG. 21. Two EDT seeds—the good seed on the left will grow only in length; the imperfect one on the right will increase in cross section as well as length.

Once a few large, uncracked crystals have been secured, it is possible to multiply their numbers by cutting them into seed plates and using a capping technique similar to that described for ADP.

Capping in this case is a slow operation. Unlike ADP the center of the cut face is not protected from circulation by the formation of tent-like corners. Plates cut from cloudy sections of a bar contain some misoriented growth and the absence of this corner protection results in a more or less flat, even growth over the whole surface, but often defective

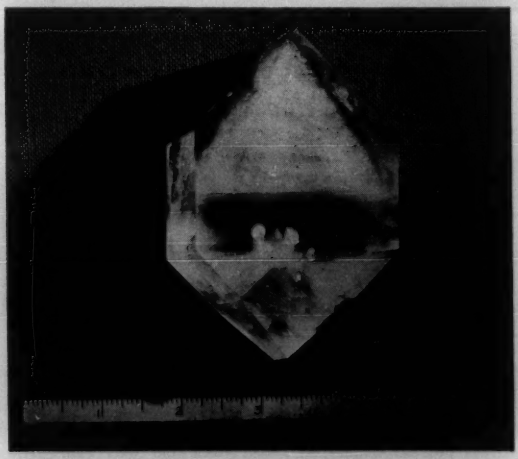

FIG. 22. Capped EDT plate seed.

due to the misoriented underlying surface. It takes about one month to produce a capped seed plate like that shown in Fig. 22. Lengthwise growth along the prism faces is appreciable.

Figure 23 illustrates the useful cuts of an EDT crystal. The clear triangular end, containing the $\{1\bar{1}0\}$ growing faces, performs the same

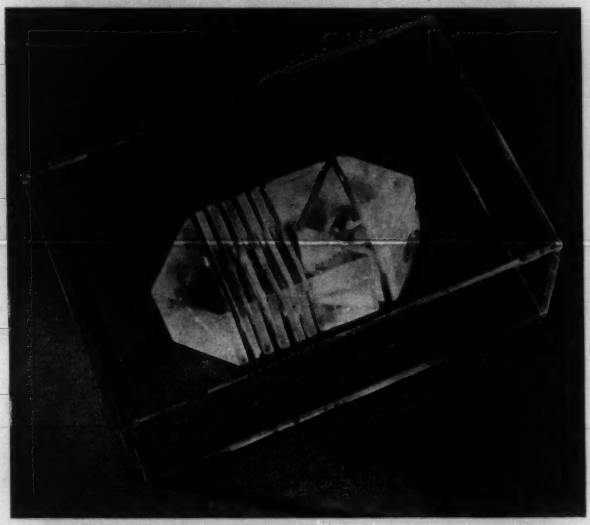

FIG. 23. Useful cuts of EDT bar.

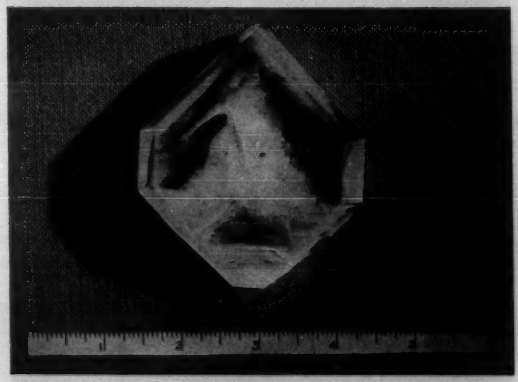

FIG. 24. Capped EDT seed grown from "butt" end.

function as the capped seed plate of ADP. The inner cap from which the bar was grown, performs the same function. The thin plates composing the clear section of the bar are used primarily for piezoelectric purposes, although in an emergency they make admirable seed plates. Any plate seed forms the growing end faces $\{1\bar{1}0\}$ on one side and the $\{110\}$ faces on the other side. Once the $\{110\}$ faces have been formed growth stops on this end of the crystal. This so-called "butt" end of the bar, in Fig. 23, has little value, although when the need is great it can be used as a seed, forming a capped unit like that shown in Fig. 24. The fish-tail section in Fig. 23 is waste; it may be dissolved and used in a new growing solution.

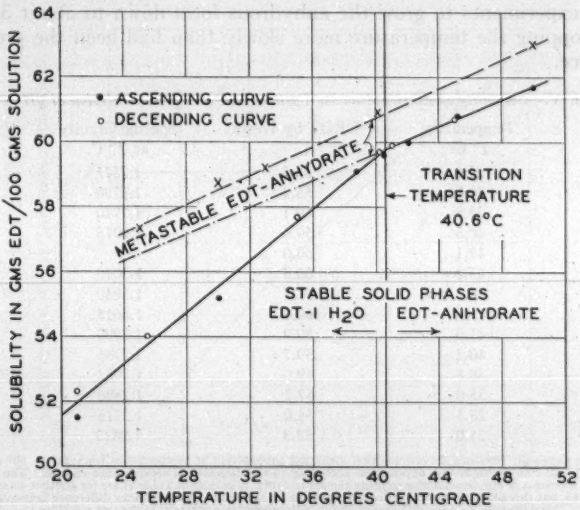

FIG. 25. Solubility relations of EDT.

EDT Monohydrate

Initially EDT crystals were grown in these Laboratories by the dropping temperature procedure over the whole range from 60° C., down to about 16° C. For nearly two years there was no evidence to suggest that any other form of EDT existed save the anhydrous monoclinic crystal. Then a report came from the commercial installation at the Western Electric Co., that an unknown crystal of different form was making its appearance in the EDT growing tanks. This new crystal of orthorhombic form grew rapidly and adhered to all faces of the EDT bars and threatened to disrupt the crystal program. Within a very

short time the same condition was observed in the pilot plant at the Bell Telephone Laboratories, many miles away, and it is now known that this "infection" was brought there on samples from the commercial installation. Once present in a plant it is difficult if not impossible to eliminate it.

A systematic investigation (11) soon made it clear that the new crystal was the monohydrate of EDT. This crystal has little if any piezoelectric response and is the stable form of EDT in contact with the saturated solution below 40.6° C., in accordance with the phase relation in Fig. 25.

Once the hydrate made its appearance, the dropping temperature method was no longer useful below 40° C. It was possible, however, in some experiments to grow the anhydrous form down to about 32° C., by dropping the temperature more slowly than had been the previous practice.

TABLE V.—*Solubility of Ethylenediamine Tartrate Salt in Aqueous Solution at pH = 7.18*

Temperature T° C.	% EDT by Weight	Specific Gravity at T° C.
21.0	51.5	1.2575
30.0	55.2	1.2780
38.8	59.1	1.2940
40.5	59.6	1.2975
42.1	60.0	—
45.2	60.8	1.3020
50.0	61.7	1.3040
45.0	60.7	1.3015
41.0	59.9	1.2990
40.2	59.7	1.2985
38.8	59.0	1.2952
35.0	57.7	1.2890
25.5	54.0	1.2715
21.0	52.3	1.2625

These data were obtained on one solution saturated successively at temperatures from 21° to 50° and then back to 21° C. Equilibrium was approached according to the direction of temperature change. The solution contained an excess of ethylenediamine to bring the pH to 7.18. The data in Table II are for another solution with a pH of 7.15, but this small pH variation does not account for the rather large solubility difference between the two solutions. This difference is due to different amounts of decomposition material in the two solutions.

The upper dashed line in Fig. 25 is a plot of the data in Table II. This line indicates that the solubility of anhydrous EDT can be considered as substantially linear between 25° and 50° C., before the EDT hydrate made its appearance. The data in Table V, taken after the hydrate was observed, are plotted as solid lines in Fig. 25. These data show the transition point at 40.6° C., above which the anhydrous EDT is the stable solid form. It seems reasonable to assume that the solubility of anhydrous EDT below 40° C., can be expressed as a linear extrapolation of the plot above this temperature. On this basis was drawn the dotted-dashed line in Fig. 25. Verification of this portion of the curve did not seem to be important in view of the limited objective of this development program.

If the two solutions used to secure the data in Tables II and V had been alike in all respects, these two solubility lines should coincide above 40° C. The fact that they are at different levels is explained by the presence of different amounts of decomposition products. This plot of the data shows the serious difficulty caused by the unstable character of the salt, and indicates that such data are of little value for crystal growing purposes.

The slope of the hydrate solubility curve below 40° C., plotted from the data in Table V, is much greater than that of the assumed slope of the anhydrous form. This explains the difficulty in growing EDT crystals by the dropping temperature procedure below 40° C., because of the increasing degree of supersaturation of the solution with respect

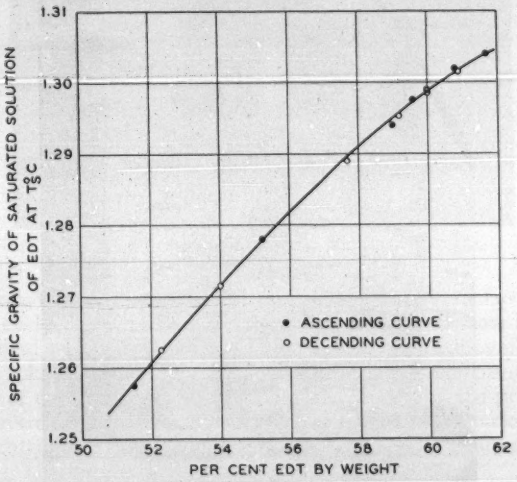

FIG. 26. Specific grvaity-solubility relation of EDT saturated solutions.

to the hydrate. At some temperature in the region of 32° C., the solution becomes so unstable that a snowstorm develops of hydrate crystals.

Figure 26 is a plot of specific gravity *vs.* solubility of EDT in saturated solutions, in accordance with the data in Table II. Obviously such a plot would not show any break at the transition point.

Spontaneous nuclei of the hydrate do not multiply at a rapid rate in the EDT solution, even at such high degrees of supersaturation as are represented by a growing temperature of 32°. This is due to the high viscosity of the solution, and the inhibiting effect of resin. Within 24 hr. after the first nuclei appear, their numbers have not increased greatly. It takes about three days to develop a first class snowstorm, such as can be produced in a few hours with ADP.

Nuclei deposit readily on the surfaces of ADP bars and thereafter induce misorientation and splinter-type growth. With EDT, the high viscosity of the solution, together with the small difference in density between growing solution (1.3) and that of the crystal (1.65) combine to prevent nuclei from readily adhering to the bar surfaces. This condition makes it possible to grow EDT crystals under more adverse conditions than are possible with ADP.

Figure 27 was taken on the third day after nucleation was first observed in an EDT solution at 33° C. By this time some of the nuclei had adhered to the faces of the bars and had grown to considerable proportions. By the next day the snowstorm of fine nuclei completely ob-

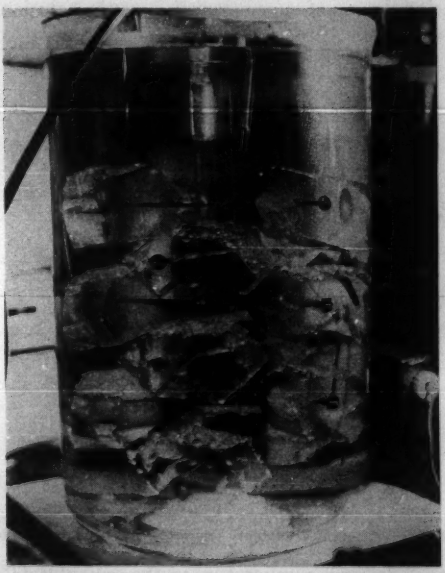

FIG. 27. Jar of EDT crystals with deposits of hydrate crystals.

scured the crystals rotating in the jar and a layer of almost two inches of salt settled to the bottom when rotation was stopped. An occasional nucleus adhering to the bar grew amazingly. See Fig. 28. The large hydrate crystal grew on the edge of the anhydrous bar in only four days. It was quite clear and transparent until removed from the jar. Thereafter the hydrate slowly lost water and turned white.

Defects and Advantages of EDT Crystals

One serious defect of the EDT crystal is its sensitivity to thermal shock, due to its rather high temperature coefficient of expansion. Un-

less the crystal is quite clear and free from misoriented areas, it will very likely crack when handled, particularly if the crystal is larger than $1\frac{1}{2}$ in. in cross-section.

FIG. 28. Single EDT crystal with large adhering hydrate crystal.

Crystals with splinter growth very frequently show cracks during the growing cycle, even at constant temperature. Such defects almost always extend into the clear end of the crystal after removal from the growing solution. This behavior suggests that misorientation, particularly of the splinter-type which induces growth on the prism faces, causes severe strains in the crystal. This sensitivity to cracking means that in cutting and processing EDT crystals, special equipment must be used. This subject has been treated elsewhere (12).

FIG. 29. Effect of impurities on EDT growth habit.

Another limitation of the EDT crystal is that it grows in only one direction along its *b* axis. ADP and quartz crystals grow equally in both directions along their long axis (*Z*-axis).

EDT crystals are surprisingly free from influences of most impurities such as are serious with ADP. Metal ions such as iron, chromium, etc., do not seem to impair the piezoelectric properties, unless these elements are present in high concentration. The amounts of calcium or magnesium present in tap water (total solids about 180 ppm.) do exert some influence on the growth habit, as may be seen in Fig. 29, right. The effect is to add $\{h\bar{k}l\}$ and $\{\bar{h}0l\}$ faces as shown. Other impurities of unknown origin, but believed to be resin products, sometimes cause the appearance of a $(0\bar{1}0)$ face normal to the *b* or *Y* axis on the growing end. See Fig. 29, left. Also infrequently the butt end has two extra faces such as those seen in Fig. 29, left.

GROWING QUARTZ CRYSTALS

Geologists have long speculated on how nature formed large, clear quartz crystals, and their efforts to solve the problem in the laboratory date back about 100 years. A comprehensive review of this work was published by Kerr and Armstrong in 1943 (13). The most notable result in this summary was that of Spezia (14) who grew about 5 mm. of clear quartz on the ends of a natural crystal. This was done in a pressure vessel in alkaline solution, in a period of six months.

After the recent war, reports were received from Germany about the Nacken process (4). Nacken worked on various aspects of crystal synthesis for more than ten years and succeeded in growing quartz by a hydrothermal method, using high temperature and pressure in alkaline solution. With Nacken's results as a starting point, the Bell Telephone Laboratories have made encouraging progress towards an understanding of the problem.

Constant Temperature Procedure

The Nacken process was based on experiments at constant temperature, and his experiments on solubility of silica led him to the conclusion that amorphous silica is ten-fold more soluble than quartz in water or alkaline solution at high temperature and pressure. Recent investigators have been unable to verify this. P. H. Egli, in a private communication, suggested that amorphous silica dissolves at a rate tremendously more rapid than the solution can tolerate for single quartz crystal growth. This subject has been treated somewhat more extensively in another paper (15).

Whatever the true explanation, Nacken showed that quartz could be grown rapidly, for a short time, under these hydrothermal conditions. Unfortunately the supersaturation increases so rapidly that quartz not only deposits on a suitable seed, but as spontaneous nuclei on all the

internal surfaces in the autoclave. Within about 24 hr. growth nearly stops. What takes place within a sealed high pressure vessel cannot be studied visually, but the growth of quartz from amorphous nutrient material may be inferred from an examination of the materials at the end of the test.

In Fig. 30 the clear quartz plate on the left gained about 0.008 in. in total thickness after 24-hr. exposure in a dilute sodium carbonate solution supersaturated with silica at conditions approximating those for the critical temperature for water: 374° C., and 3200 psi. Before the test, the quartz plate had ground, opaque surfaces, like the second from the left in Fig. 30. Initially the amorphous nutrient material was in the form of a clear cylinder of Vitreosil glass, like that shown in the photograph. Afterwards, this cylinder of silica was found to be largely con-

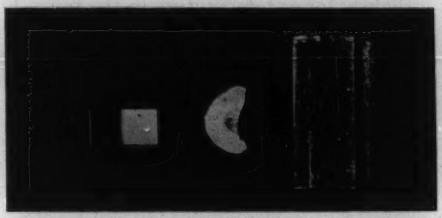

FIG. 30. From left to right: Seed plate with 0.008 in. of clear quartz growth; seed plate before growth, showing opaque surfaces; cross section of nutrient amorphous silica mostly converted to polychrystalline quartz during a run; piece of amorphous silica (Vitreosil glass) before a run.

verted to a porous mass of fine quartz needles. A cross-section of this cylinder after conversion, is shown also. Subsequent check tests showed that the total growth of 0.008 in. occurred in the first few hours.

As the autoclave approaches operating temperature, the solution becomes so supersaturated with respect to quartz that the seed grows rapidly. Spontaneous seeding also occurs, both on the walls on the container and on the surface of the nutrient material. Within a matter of a few hours, the amorphous supply becomes covered with quartz needles, the solution loses its supersaturation, and growth practically ceases on the seed. Further solution of the amorphous silica continues under the porous layer of quartz, but is immediately followed by redeposition at that point. Failure to achieve continuous growth was encountered by Nacken, and he found no solution for this problem.

Experience in the growing of water-soluble crystals shows that, in general, a 5 per cent supersaturation may be maintained without spurious seeds, but with amorphous silica as nutrient, the degree of supersaturation greatly exceeds this small percentage. This explains the uncontrolled deposition of spontaneous nuclei within the bomb.

Temperature Gradient Procedure

Initial experiments with the Nacken method were made in an autoclave only 1 in. in diameter by 1 in. deep. This was not difficult to hold at nearly constant temperature. Other experiments in the same diameter autoclaves but 12 in. in length were difficult to keep at the same temperature from top to bottom, with the autoclaves standing vertically in the furnace. The top was always somewhat cooler than the bottom. Quartz seeds in the top continued to grow slightly for several weeks, whereas in the short chamber growth practically ceased after the first day.

Knowing that the amorphous material rapidly became covered with a film of redeposited quartz, this slow growth in the longer autoclave seemed to be due to solution of crystalline quartz in the bottom and deposition at a slightly lower temperature in the top. Further experiments were made with crystalline quartz as nutrient material, making use of a temperature gradient along the vertical length of the autoclave. It is sufficient to summarize the conclusions of this intermediate stage of the investigation, as follows:

1. Clear quartz could be deposited at rates as high as 0.03 in. per day, but only for short periods of a few hours, and in the presence of much spurious seeding, using amorphous silica as the nutrient.
2. Growth at rates of a maximum of 0.001 in. per day could be sustained, apparently for a considerable period, with crystalline quartz as nutrient, and with no spurious seeds.
3. Growth at a rate of about 0.03 in. per day could be obtained with crystalline quartz as nutrient, in narrow zones within the autoclave where marked changes in solution density occurred, and this suggested that better results might be obtained at higher solution density.

Degree of Fill

To secure higher density of the solvent the autoclave was filled with more liquid than had been used previously. After some experiments it was found that the best results were obtained with the container filled to 80 per cent of its free volume with the alkaline solution, measuring this degree of fill at room temperature. The operating temperature used was 400° C., and the nutrient material was clear Brazilian quartz scrap.

Nacken considered that there might be some transfer of silica through the vapor phase under the hydrothermal conditions he employed. The results with 80 per cent filling makes it unnecessary to consider vapor transfer. The autoclave becomes filled with the solution long before the critical temperature is reached, and at 400° C., under these conditions, it is more appropriate to consider the solution as having a density of 0.8, neglecting the slight changes in container volume.

The solubility of silica is a function of the density of the continuous phase, whether liquid or vapor, according to Kennedy (16), and of course it depends also upon the alkalinity of the solution. The degree of supersaturation, with respect to quartz, the stable solid phase at this temperature, is obtained by establishing a suitable temperature difference between the bottom and top of the autoclave.

Application of the Temperature Gradient Method

Preliminary Tests.—Initial experiments at these higher pressures were with a simple type of welded autoclave, consisting of a 1-in. I. D. by ½-in. wall, seamless steel tube, about 12 in. long. Steel plugs were welded into each end. This design was first suggested by the Brush Development Co. Several crystals of the size shown in Fig. 31, middle

FIG. 31. From left to right: quartz seed plate; grown crystal; larger quartz crystal grown in leaky autoclave.

sample, were grown in this kind of autoclave, from seed plates such as that shown at the left. The seed plate is a CT cut, that is, cut nearly parallel to one of the minor rhombohedral faces of the quartz crystal. Because these minor rhombohedral faces grow most rapidly, such plates are most suitable for growing experiments.

The center crystal in Fig. 31 is viewed nearly along the major surfaces of the seed plate, so that some idea of the thickness of growth may be obtained. The seed plate was 0.035 in. thick and the added thickness shown by the grown crystal was 0.15 in. in the ten day growing period. This corresponds to a rate of 0.015 in. per day, about 15 times as fast as had been secured in tests made at lower pressures with quartz as nutrient material.

In another autoclave in this same run, the crystal shown on the right in Fig. 31 was grown. This crystal was grown from a half-cap seed,

½ in. long, and it may be seen as the white, cloudy portion just to the left of the center of the bar. The total length of this crystal was 1½ in., corresponding to a growth rate of 0.1 in. per day. This result marked an important milestone in the investigation of the growth of quartz in the laboratory. For the first time it was shown that with suitable conditions, clear quartz crystals could be grown at a remarkably high rate.

The capped end of the seed grew quite clear, all during the run. The other end, initially a face cut at right angles to the long axis of the crystal, grew rapidly at first with some cloudy occlusions, reminiscent of the way in which ADP crystals grow from *Z*-cut seed plates. Only the three major rhombohedral faces remained on the ends of this

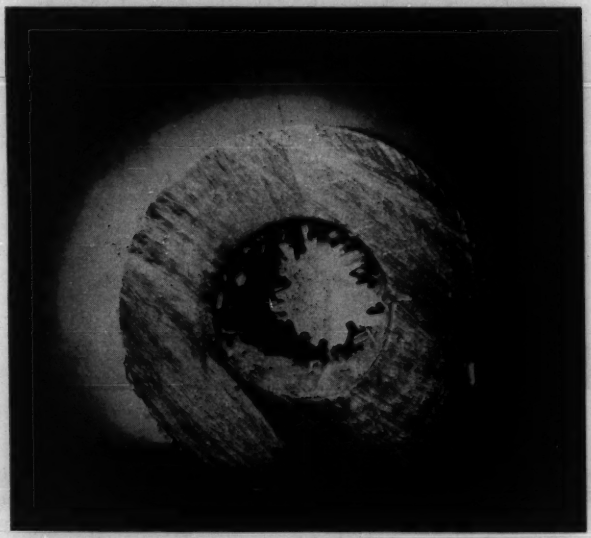

FIG. 32. Section of leaky autoclave showing spontaneous quartz crystals.

crystal. This fact, at the time considered unimportant, has since been found to be the usual habit of these synthetic quartz crystals. Whether this is because the laboratory product is grown much faster than in nature, and thus develops this habit, or because of the presence or absence of certain habit-modifying impurities is unknown. The answer awaits further investigation.

In this test, the reason for the rapid growth was found in the fact that the autoclave leaked at the top weld. The leak was very slight, only a small fraction of the liquid was lost during ten days, an unusual

fact in itself. This leak had the effect of giving a somewhat greater temperature drop in the vicinity of the seed. This caused a greater degree of supersaturation at this point and consequently an increased growth rate on the seed located near the top of the autoclave. The degree of supersaturation was so high that spontaneous crystallization occurred over all the internal surfaces. Some of these needles are shown on the support wire in Fig. 31, and the inside of the autoclave is shown in section in Fig. 32. It looks much like the inside of a natural geode or vug, in which clear quartz crystals are frequently found in nature.

This experiment suggests that crystal deposits in such geodes were the result of steam leaks through cracks in the earth's surface, not a new idea of course to geologists. This steam was initially at high temperature and pressure well below the surface and had dissolved appreciable amounts of silica, a fact perhaps not so well recognized. As the steam expanded through the cracks it gave up the dissolved silica due to changing density conditions at a much more rapid rate than would have been possible by liquid solution and deposition at lower temperatures. The rapid growth of quartz in this experiment suggests also that some of the quartz found in nature may not have required thousands of years to grow, but may have been produced in relatively short periods.

Autoclave Design

The welded steel tube autoclaves just described had some limitations. One was that the unit must be sawed open after a run, a destructive operation. Also it is very difficult to avoid leaks in the welding operation on such a tube. The pressures developed by an 80 per cent fill, at 400° C., are of the order of 15,000 psi. and it is extremely difficult to prevent leaks at such pressures and temperatures in gasket-sealed autoclaves. Extensive experiments led to the development of a welded liner design as shown in Fig. 33. This consists of a thin-walled steel tube, or liner, enclosed in a heavy-walled outer container. Each end of the liner tube is closed with a cup. The rim of each cup, flush with the end of the liner, is welded with an atomic hydrogen flame, a relatively simple operation. The sealed liner contains the ingredients necessary for growing quartz crystals. These are: a supply of nutrient quartz in the bottom; sufficient alkali solution to fill the free volume of the tube to 80 per cent of its capacity at room temperature; and one or more quartz seeds mounted on an iron wire frame above the nutrient material. The liner is slipped into a close-fitting outer shell.

Into each recessed end of the liner is fitted a retainer cap, shown in section in Fig. 33. The difference in length of shell and liner permits clearances between the flanges on the retainer caps, and the ends of the outer shell so that the screw caps can apply enough force to fit the caps down snugly over the welded rims. The taper groove prevents spreading of the weld. The pressure of the expanding liquid at the operating

temperature distends the liner so that it fits tightly within the retaining walls at all points, much as though it were a rubber balloon.

The liners are of inexpensive low carbon seamless steel tubing and they are discarded after each run. The top cup is machined to a thickness of $\frac{1}{32}$ in. where it makes contact with the hole in the top retainer cap. This forms a safety diaphragm. The hole is 0.0938 in. in diameter. This together with the cup thickness makes a diaphragm which will withstand a pressure of about 20,000 psi. Failure of the safety

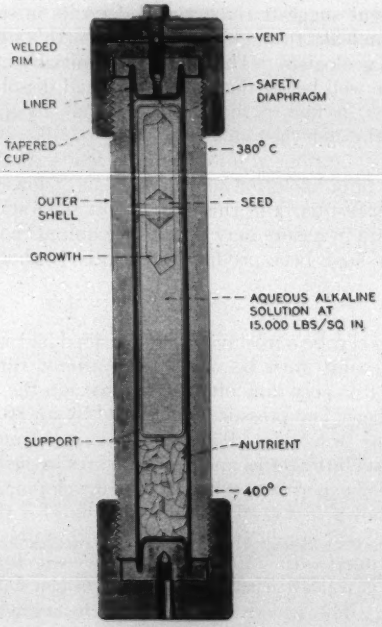

FIG. 33. Sketch of liner type autoclave for high pressure temperature gradient method of growing quartz.

diaphragm at the operating condition causes no damage with the 1-in. I. D. autoclaves, in a properly constructed furnace, such as that shown in Fig. 34.

Ordinary cold-rolled seamless steel tubing with $\frac{1}{2}$-in. wall serves for the 1-in. I. D. autoclaves. A 2 per cent manganese steel container with a 2-in. I. D. opening and a 1-in. wall showed no creep under these conditions. Three-in. I. D. autoclaves with $1\frac{1}{2}$-in. wall, made of 304 stainless steel gave 0.072-in. creep in the 3-in. diameter in less than 3000 hr., and

were not satisfactory. Autoclaves of 316 stainless steel of 3-in. bore and 1½-in. wall have been used and have shown no creep.

The filled autoclave is set on a metal hot plate in the furnace, and completely surrounded with vermiculite insulation. The hot plate is heated to about 450° C., and the temperature at the top of the nutrient layer of quartz is then at about 400° C., as measured by a thermocouple strapped to the outside of the autoclave at this level, just above the bottom cap. Another thermocouple, mounted just below the upper threaded cap is maintained at a temperature from 10° to 20° below the lower one, by adjustment of the amount of insulation on top.

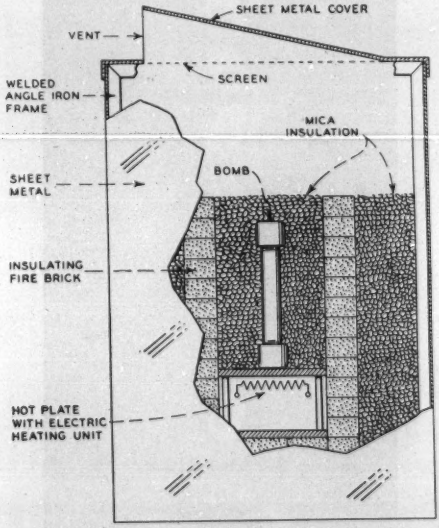

FIG. 34. Furnace for use with temperature gradient method of growing quartz.

Autoclave Operation

Quartz dissolves in the bottom until the alkaline solution is quite saturated with silica at some temperature representing the effective value in the nutrient layer. Convection currents are sufficient to carry this saturated solution to the top of the chamber at a rate more than sufficient to transfer all the silica needed for high growth rates. Because the top zone in the autoclave is at a lower temperature than the bottom, the solution becomes supersaturated with respect to quartz and silica deposits uniformly on the seed. The process continues as long as nutrient material is available for solution in the bottom of the autoclave. The rate of growth is determined by the temperature differ-

ence maintained between the nutrient material in the bottom and the solution surrounding the seed near the top. The thermocouple measurements are considered as reliably indicating the temperature of the solution at these points, although it is impossible to measure the actual solution temperature with any equipment now available.

The threaded caps are usually so firmly bound by the internal expansion force, that they are difficult to unscrew after a run. If forced, the threads may break or wedge; then the cap must be sawed off and

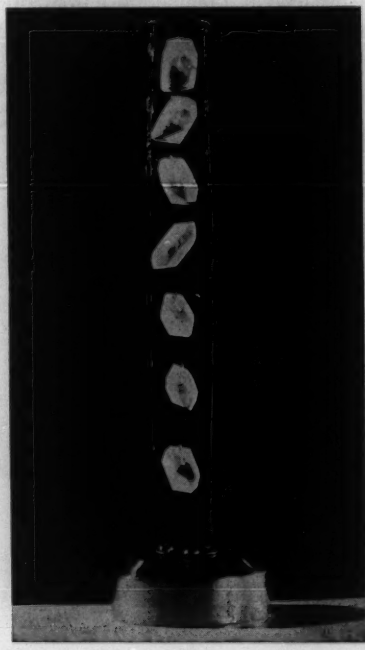

FIG. 35. Series of quartz crystals grown at one time in a single autoclave.

destroyed. A simple solution of this problem is to apply a compressive force of from 40 to 80 tons to the ends of the autoclave. This pressure cracks loose the threads, which are designed with ample clearance, by compressing the liner slightly. The caps then unscrew readily by hand.

Progress to Date in the Growth of Quartz

Much remained to be done after it was found that quartz crystals could be grown at rates of the order of from 0.05 in. to 0.1 in. per day.

The story of this third phase of the development work is still incomplete, but the progress made in this research may be illustrated by some examples.

Figure 35 shows that as many as 7 quartz crystals can be grown in a single autoclave in one experiment. In this case the top crystal grew to a length of 1 in. in 10 days, again verifying the rate observed in the leaky autoclave experiment. Growth was uncontrolled in that some spontaneous nuclei were found on the frame and on the walls of the liner. The incidence of such unwanted seeds was much less than in the leaky autoclave test.

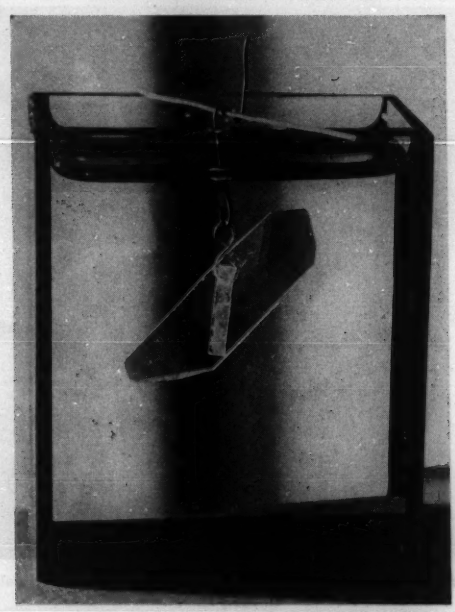

FIG. 36. Top crystal from Fig. 35, immersed in oil of like index of refraction—to show absence of imperfections.

The top crystal in this run is again shown in Fig. 36. It is immersed in oil having the same index of refraction as quartz and this picture shows that the cloudy outer surface of the crystal was only a superficial etching. The interior is quite clear and free from misorientation. Even this surface cloudiness can be eliminated by the proper choice of alkaline solution. Figure 37 shows 8 crystals grown in a single run.

In this case the growth was so clear that the crystals resemble

jewels. These were grown at a rate of 0.06 in. per day and showed no evidence of lack of control for there were no spurious seeds.

Figure 38 is a picture of some of the largest quartz crystals which have been grown in the laboratory up to the present time. These three were grown in one month in a single autoclave. The top crystal weighs 312 grams, and the other two 144 and 72, respectively. The cloudy appearance of some of the surfaces is due to deposits of sodium silicate at the end of the run. This surface effect occurs only infrequently and is not difficult to overcome.

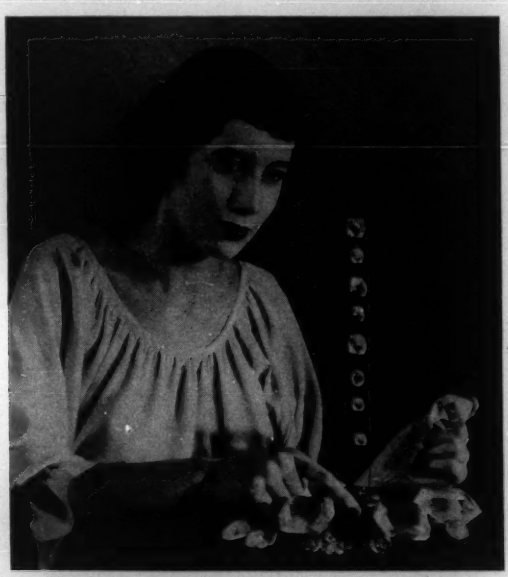

FIG. 37. Series of 8 quartz crystals grown at one time—also several large crystals, one weighing about ¼ lb.

Figure 39 is of interest as a display of the progress made in four years of study. The thin transparent plate on the left grew 0.006 in. in thickness in less than one day. The large crystal on the right (top one in Fig. 38) grew 1 in. in thickness in 30 days and increased from 36 grams to 312 grams in weight. This photograph emphasizes the great advance made in the rapid growth of quartz crystals of useful size, which geologists had believed might have required long periods of time to form in nature.

SUMMARY

This paper presents a discussion of the processes for growing three important piezoelectric crystals. Of these, two have been produced commercially and have found wide use in the electrical industry. The third process, that of imitating nature in the growing of quartz, is shown to be feasible. It should be evident from the discussion that the number of unsolved problems in the art of growing large single crystals greatly exceeds those for which answers have been found. Despite this it is considered that much has been accomplished in a relatively few years. The techniques which have been developed in the course of this work should be useful in other branches of the subject, and it is for this reason that the paper contains some detailed discussion of certain problems.

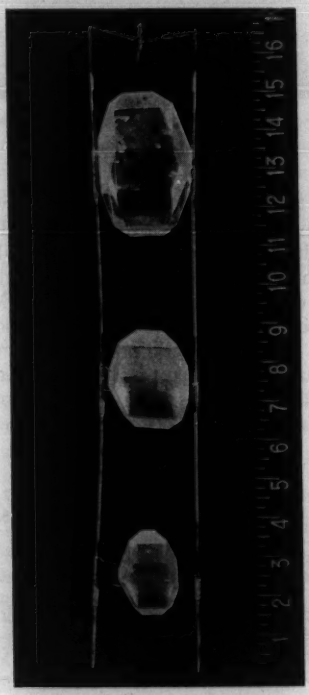

FIG. 38. Three quartz crystals grown to an added weight of 1 lb. in 30 days. Largest crystal weighs 321 grams.

FIG. 39. Four years of quartz growing research.

Acknowledgment

It would not be fitting to conclude this survey without emphasizing the fact that this development was a cooperative effort, involving many contributions by others. Particular mention should be made of valuable engineering contributions by C. J. Christensen, A. N. Holden and E. Buehler; advice on crystallography by W. P. Mason, W. L. Bond and Mrs. E. A. Wood; encouragement and guidance by S. O. Morgan and G. T. Kohman. The list would not be complete without reference to vigilance on the part of a group of night plant watch operators, whose constant personal interest in this work was an important factor in the dependable continuous operation of the crystal growing equipment.

REFERENCES

(1) A. M. NICHOLSON, *Trans. Am. Inst. Elect. Eng.*, Vol. 38, p. 1467 (1919).
(2) A. M. NICHOLSON, U. S. Patents: Nos. 1,495,429; 1,525,823; 1,655,625; 2,135,852; 2,212,845.
(3) G. BUSCH, *Helv. Phys. Acta*, Vol. 11, pp. 269–298 (1938).
(4) R. NACKEN, "Report on the Research Contract for Synthesis of Oscillator Crystals," Captured German Report RDRDC/13/18. Also—Office of Technical Services Reports, PB-6498, PB-18,784, PB-28,897, and others, Dept. of Commerce, Washington, D. C.
(5) A. C. WALKER, *Bell Laboratories Record*, Vol. 25, p. 357 (1947).
(6) A. C. WALKER AND G. T. KOHMAN, *Trans. Am. Inst. Elect. Eng.*, Vol. 67, Part 1, p. 565 (1948).
(7) G. H. BUCHANAN AND G. B. WINNER, *J. Ind. and Eng. Chem.*, Vol. 12, p. 448 (1920).
(8) H. J. KOLB AND J. J. COMER, *J. Am. Chem. Soc.*, Vol. 67, p. 894 (1945).
(9) A. N. HOLDEN, *Discussions of the Faraday Soc.*, No. 5, p. 312 (1949). Also U. S. Patent 2,484,829 (1949).
(10) C. J. CHRISTENSEN AND A. C. WALKER, U. S. Patent, 2,459,869.
(11) G. T. KOHMAN, *Bell Laboratories Record*, Vol. 28, p. 13 (1950).
(12) J. P. GRIFFIN AND E. S. PENNELL, *Trans. Am. Inst. Elect. Eng.*, Vol. 67 (1948).
(13) P. F. KERR AND E. ARMSTRONG, *Bull. Geo. Soc. Amer.*, Vol. 54, Supp. 1, pp. 1–34, (1943).
(14) G. SPEZIA, *Acc. de Scienze di Torino*, Vol. 44, p. 95 (1908).
(15) A. C. WALKER AND E. BUEHLER, *J. Ind. and Eng. Chem.*, Vol. 42, p. 1369 (1950).
(16) G. C. KENNEDY, *Econ. Geol.*, Vol. 39, p. 25 (1944).

CARRIER COMPENSATION FOR SERVOMECHANISMS [1]

BY

H. ELMORE BLANTON [2]

Part II [3]

4. R-C NETWORKS

Although R-C networks may be considered as a special case of R-L-C networks where all inductances are equal to zero, they have received sufficiently wide application in the field of carrier compensation to merit separate treatment in this paper. Due to the several distinct advantages of R-C networks, the use of such networks has been investigated by several groups and to the author's knowledge there is much more material published in regard to the use of R-C networks in carrier compensation than there is published concerning the use of the more general R-L-C class of networks.

4.1 Transfer Characteristics Obtainable

The types of transfer characteristics obtainable with R-C networks are much more restricted than in the general R-L-C case. Considerable study has been given to the use of R-C circuits to obtain carrier-frequency lead networks (3, 7, 10, 11)[4], and special types of circuits have been adapted for this purpose. These circuits will be discussed in the later sections of this chapter.

On the basis of general circuit synthesis techniques, evidence (12) can be given that any transfer characteristic can be realized with an R-C network provided that the corresponding admittance function,

$$Y_{12}(\lambda) = \frac{P(\lambda)}{Q(\lambda)} = -\frac{E_2}{E_1} \quad \text{(when network is normalized for 1 ohm load),}$$

is the ratio of two polynomials, $P(\lambda)$ and $Q(\lambda)$, which are of the same degree and which have all of their zeros on the negative real axis. The zeros must be simple and those of $P(\lambda)$ must alternate with those of $Q(\lambda)$. The absolute value of the smallest root of $P(\lambda)$ must be greater than the absolute value of the smallest root of $Q(\lambda)$.

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[2] Department of Electrical Engineering, Massachusetts Institute of Technology, Cambridge, Mass.

[3] Part I appeared in this JOURNAL for November, 1950, p. 391.

[4] The boldface numbers in parentheses refer to the references appended to this paper.

4.2 Types of Networks Employed

Three R-C network structures have been found particularly adapted to the realization of the transfer characteristics required for carrier compensation. They are:

1. The parallel-T network.
2. The bridge-T network.
3. The lattice network or Wein bridge.

These structures are illustrated in Fig. 7 for the case of networks designed to give carrier-frequency lead compensation. In addition to these structures, networks may be realized in the general ladder structure for many of the desired compensation transfer characteristics.

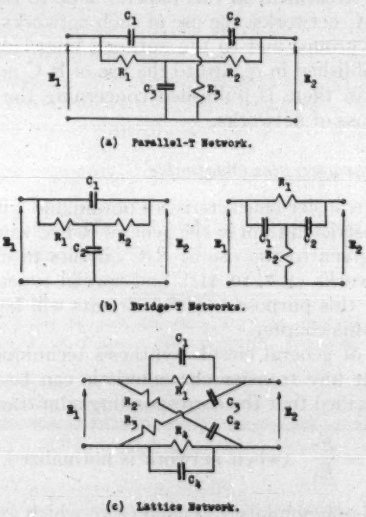

FIG. 7. Carrier-compensation lead networks.

As the lattice structure may be used only where the electrical circuit is balanced to ground, most of the research on carrier-compensation networks has been concentrated on the first two types of structures which may be used where one side of the electrical circuit is ground. The parallel-T network is of particular importance and will be discussed further in the next section.

The general ladder structure presents certain advantages, especially insofar as allowable tolerance (13, pp. 379, 380) on element values is

concerned; however, as a large number of elements is usually required to realize a given transfer characteristic the ladder structure is to be avoided in many applications.

4.3 Further Comments on the Parallel-T Network

The parallel-T network is perhaps the most used R-C carrier-compensation network structure, and like many of the circuits used in servo design, it was adapted from an idea first presented in another field of electronics. Since that time much study has been devoted to the evaluation of its characteristics and development of design formulae applicable to the servo field.

Augustadt (11, p. 167) in his application of May 23, 1936, for United States Patent Number 2,106,785 described the parallel-T network and suggested the use of the circuit as a power supply filter. Scott (14) in 1938 further considered the circuit and outlined the advantages of the frequency-selective nature of the response. He suggested using the circuit in the feedback path of an amplifier to improve the selectivity of the amplifier, and briefly discussed the application to oscillators and harmonic analyzers. Others (15, 16, 17, 18) have investigated other aspects of the circuit, considering its application to the field of frequency-selective measurements and to electronic equipment which requires very sharp resonance effects at certain frequencies.

The first extensive publication concerning the use of the parallel-T network in the field of carrier compensation was by Sobczyk (7). Based on work in the Radiation Laboratory of the Massachusetts Institute of Technology during World War II, this publication represents an important advancement in the study of both the parallel-T network and the field of carrier compensation in general. The author considers the use of the parallel-T network in a servo employing a two-phase motor and proceeds to develop formulae which express the frequency response of a parallel-T lead network to a modulated-carrier signal. These formulae are stated in the terminology normally employed in the servo field; hence, they are useful for a wide range of compensation problems. The employment of the parallel-T network for a carrier compensation circuit is briefly mentioned in several servo textbooks (10, 11), and the application of the circuit to eliminate hunting in a system which includes a two-phase motor is discussed by Larsen (3). Several other papers (6, 19, 20, 21) are also available on various aspects of the subject.

The response of parallel-T lead network (7) may be expressed as

$$\frac{E_2}{E_1}(j\omega) = \frac{L}{T_d} \times \frac{1 + jT_d \dfrac{\omega + \omega_c}{2}(\omega - \omega_c)}{1 + jL \dfrac{\omega + \omega_c}{2}(\omega - \omega_c)},$$

where

ω_c = the carrier frequency,
T_d = the derivative time constant, and
L = the lag time constant.

This response function is a good approximation to the desired carrier-frequency lead-compensation function over a narrow band of frequencies in the vicinity of the carrier frequency provided the lag time constant is several times smaller than the derivative time constant. By virtue of the network design, this provision can be fulfilled as the lag time constant must be less than the reciprocal of the carrier frequency, whereas values of $T_d\omega_c$ of from 5 to 50 have been found most useful in servo compensation.

The parallel-T network is sensitive to loading, but several interesting variations in values permit the design of networks suitable for the various applications. In the most general solution a parallel-T lead network may be designed for any specified input or output impedance. Other formulae have been developed which permit the design of a network with the same input and output impedances, or of one with arbitrarily specified capacitances or resistances. The latter variation is particularly useful in some applications, for instance where only a limited range of values of capacitance is available for use, or where a network with variable parameters is desired and variable capacitances are not available. Newton (6) has suggested the use of a set of design equations which result in a network containing two equal resistors, two equal capacitances, a variable resistor, and a variable capacitance. With this circuit the carrier frequency determines the fixed elements, and the variable elements permit a controlled variation in the derivative and lag time constants of the network.

If one of the shunt elements of the parallel-T network is removed from the circuit the resulting network is called a bridge-T network. The bridge-T network has also proven useful in carrier compensation, and can be used instead of the parallel-T network for certain applications. The greatest disadvantage of the bridge-T lead network is that the attenuation in the vicinity of the carrier frequency is much greater than with a parallel-T lead network; therefore, more electronic gain must be provided in order to obtain the proper gain for the carrier frequency. As a result, the higher frequencies, such as the unwanted harmonics of the carrier frequency, will be more highly amplified and perhaps cause distortion in the output. The natural advantage of the bridge-T network, as compared with the parallel-T network, is that fewer elements are required.

One important consideration in network synthesis is the tolerance allowable on element values in order to obtain a certain degree of accuracy in the transfer characteristic. In the parallel-T lead network (7), if a uniform tolerance of ± 1 per cent is allowed on element values,

the resonant frequency of the network will be within ± 6 per cent of the correct value, and the frequency indicated by $1/\pi T_d$ will be within ± 1.65 per cent of the carrier frequency of its correct value; however, a tolerance of about ± 0.57 per cent is required to maintain less than $\pm 10°$ phase shift at the carrier frequency.

4.4 *Advantages of R-C Networks*

R-C networks have several advantages in carrier-compensation systems. Resistances and capacitances are usually readily obtainable in a wide variety of values and tolerances; therefore, the required networks may be easily synthesized. In most applications commercially available resistances and capacitances may be considered as pure elements, and the circuits in which they are employed may be expected to give the desired response within the limits dictated by tolerances on element values. Resistances and capacitances are available which vary very little with time or temperature; hence, networks constructed with them are quite stable and relatively unchanging in response characteristics. The fact that these elements may be obtained in very small lightweight units is a deciding factor in certain cases.

4.5 *Disadvantages of R-C Networks*

The major disadvantage of R-C networks, or perhaps, better stated, the major limitation on the use of R-C networks is the small variety of transfer characteristics which can be easily synthesized. The parallel-T and bridge-T networks in their simplest form are readily adaptable only to carrier lead compensation, and although a wider variety of transfer characteristics is available through the use of the generalized R-C network synthesis procedures, the resulting networks are often quite complex and contain a prohibitively large number of elements for applications where simplicity of design is an advantage.

5. OTHER METHODS OF CARRIER COMPENSATION

Passive electrical networks, as discussed in Chapters 3 and 4, have received wide application in the field of carrier compensation; however, there are other methods of carrier compensation which have been employed successfully. The method of demodulating, compensating, and remodulating the electrical signals has been used for some time, and during recent years additional study has been given to the possibility of obtaining the desired results by electromechanical means rather than solely by electrical means. Various electromechanical devices have found wide usage as feedback devices in carrier servo systems to produce functions which are helpful in system compensation. The problem of carrier-frequency instability has been minimized by a method which introduces phase lead which is independent of the carrier fre-

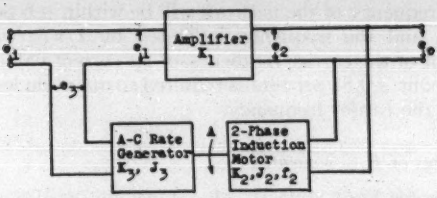

FIG. 8. Electromechanical lead-compensation device No. 1.

(Copied from Ref. 23.)

In this circuit:

$e_o = e_2 = K_1 e_1$ — e_i, e_1, etc. are suppressed-carrier modulated signals. A primed variable denotes the envelope of the variable.

$e_1 = e_i - e_3$

$K_2 e_3' = [(J_2 + J_3)p^2 + f_2 p]A$, where $p = \frac{d}{dt}$

$\omega_i \ll \omega_c$

$e_2 = K_2 p A$

$$\frac{e_o'}{e_i'} = \frac{K_1 f_2}{f_2 + K_1 K_2 K_3} \times \frac{1 + p\left(\frac{J_2 + J_3}{f_2}\right)}{1 + p\left(\frac{J_2 + J_3}{f_2 + K_1 K_2 K_3}\right)}$$

Let: $\frac{f_2 + K_1 K_2 K_3}{f_2} = r$ $\qquad \frac{J_2 + J_3}{f_2} = T$

$$\frac{e_o'}{e_i'} = \frac{K_1}{r} \times \frac{1 + pT}{1 + p\frac{T}{r}}$$

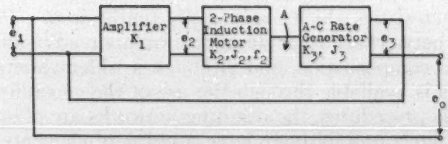

FIG. 9. Electromechanical lead-compensation device No. 2.

(Copied from Ref. 23.)

In this circuit:

$e_2 = K_1 e_i$ — e_i, e_2, etc. are suppressed-carrier modulated signals. A primed variable denotes the envelope of the variable.

$e_o = e_i - e_3$

$K_2 e_3' = [(J_2 + J_3)p^2 + f_2 p]A$, where $p = \frac{d}{dt}$

$\omega_i \ll \omega_c$

$e_3 = K_2 p A$

$$\frac{e_o'}{e_i'} = \frac{f_2 - K_1 K_2 K_3}{f_2} \times \frac{1 + p\left(\frac{J_2 + J_3}{f_2 - K_1 K_2 K_3}\right)}{1 + p\left(\frac{J_2 + J_3}{f_2}\right)}$$

Let: $\frac{f_2}{f_2 - K_1 K_2 K_3} = r$ $\qquad \frac{J_2 + J_3}{f_2 - K_1 K_2 K_3} = T$

$$\frac{e_o'}{e_i'} = \frac{1}{r} \times \frac{1 + pT}{1 + p\frac{T}{r}}$$

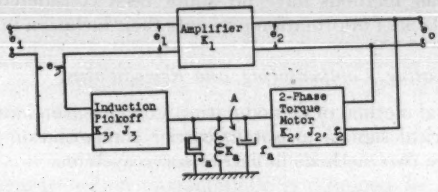

FIG. 10. Electromechanical lead-compensation device No. 3.

(Copied from Ref. 23.)

In this circuit:

$e_o = e_2 = K_1 e_1$
$e_1 = e_i - e_3$
$e_3 = K_3 A$

e_i, e_3, etc. are suppressed-carrier modulated signals. A primed variable denotes the envelope of the variable.

$$K_2 e_2' = [(J_2 + J_3 + J_a)p^2 + (f_2 + f_a)p + K_a]A, \quad \text{where } p = \frac{d}{dt}$$

$w_d \ll w_c$

Let: $J_2 + J_3 + J_a = J$, $f_2 + f_a = f$, $K_a + K_1K_2K_3 = K$

$$\frac{e_o'}{e_i'} = K_1 \times \frac{Jp^2 + fp + K_a}{Jp^2 + fp + K}$$

Let: $p = jw$ $\quad \frac{K_a}{J} = w_n^2$ $\quad \frac{w}{w_n} = u$

$$\frac{f}{2\sqrt{K_a J}} = \zeta \qquad \frac{K}{K_a} = h^2$$

$$\frac{e_o'}{e_i'} = \frac{K_1}{h^2} \times \frac{-u^2 + j2\zeta u + 1}{-\left(\frac{u}{h}\right)^2 + j2\left(\frac{\zeta}{h}\right)\left(\frac{u}{h}\right) + 1}$$

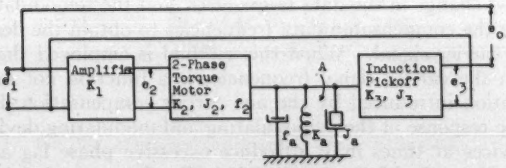

FIG. 11. Electromechanical lead-compensation device No. 4.

(Copied from Ref. 23.)

In this circuit:

$e_o = e_2 - e_3$
$e_2 = K_1 e_i$
$e_3 = K_3 A$

e_i, e_3, etc. are suppressed-carrier modulated signals. A primed variable denotes the envelope of the variable.

$$K_2 e_2' = [(J_2 + J_3 + J_a)p^2 + (f_2 + f_a)p + K_a]A, \quad \text{where } p = \frac{d}{dt}$$

$w_d \ll w_c$

Let: $J_2 + J_3 + J_a = J$, $f_2 + f_a = f$, $K_a - K_1K_2K_3 = K$

$$\frac{e_o'}{e_i'} = \frac{Jp^2 + fp + K}{Jp^2 + fp + K_a}$$

Let: $p = jw$ $\quad \frac{K_a}{J} = w_n^2$ $\quad \frac{w}{w_n} = u$

$$\frac{f}{2\sqrt{K_a J}} = \zeta \qquad \frac{K_a}{K} = q^2$$

$$\frac{e_o'}{e_i'} = \frac{1}{q^2} \times \frac{-(qu)^2 + j2(q\zeta)(qu) + 1}{-u^2 + j2\zeta u + 1}$$

quency. Other methods have no doubt been considered by research workers, but lack of information prevents their inclusion in this chapter.

5.1 Demodulating, Compensating, and Remodulating

The general method of demodulating, compensating, and remodulating the electrical signals to obtain carrier compensation has been accomplished by two methods in carrier servo systems:

1. By the use of electronic demodulators and modulators, and non-carrier methods of compensation.
2. By the use of electromechanical demodulators and modulators, and mechanical methods of compensation.

The first method consists of the application of readily available information on electronic components and non-carrier compensation techniques and consequently merits only brief consideration in this paper; however, the second method, utilizing electromechanical devices, has been developed specifically for carrier servo systems and will be discussed in more detail.

Certain considerations are basic to all compensation systems which employ the scheme of demodulating, compensating, and remodulating the electrical signals. The sequence of events is the demodulation of the carrier signal to obtain a signal which consists only of the data frequencies, the application of appropriate compensation devices to obtain the desired change in the data frequencies, and the remodulation of the carrier by the compensated data frequencies to obtain the desired compensated carrier signal. When this method is employed the resulting change in the carrier signal frequencies is a function not only of the compensation introduced by the non-carrier compensation device, but also of the response of the demodulating and modulating devices. The latter devices at times may introduce excessive phase lag and render this method unsuitable for particular applications.

A variation in this method of compensation is to place the demodulating, compensating, and remodulating devices in the feedback path of a high-gain electronic amplifier. As a result of the familiar feedback principle (22, p. 33), the over-all response of this system will be approximately the reciprocal of the transfer characteristic of the feedback path. This method can be employed very successfully to obtain transfer characteristics, which otherwise could not be easily obtained.

The best available description in the case of electromechanical compensation devices is contained in a report which was published by McDonald (23) of University of Michigan in October 1948. He describes several such devices and considers methods of employing them to obtain carrier lead and integral compensation. (See Figs. 8–12.) In Figs. 8–11 the circuits are arranged to obtain carrier lead compensation, as

may be noted by the equations derived in each figure. By arranging the circuits shown in Figs. 8 and 10 so that the feedback is regenerative instead of degenerative, and by arranging the circuits shown in Figs. 9 and 11 to add the voltages e_i and e_3 instead of subtracting them, the resulting transfer characteristics, e_o'/e_i', will give integral compensation rather than lead compensation.

In electromechanical devices of the types illustrated the motor acts as a demodulator and the generator or induction pickoff acts as a modulator. The compensation characteristic is obtained by adjusting the variable parameters of the device. The motor-generator combination of Figs. 8 and 9 leaves little latitude for adjustment, but the characteristics of the systems of Figs. 10 and 11 are subject to wide variation. A

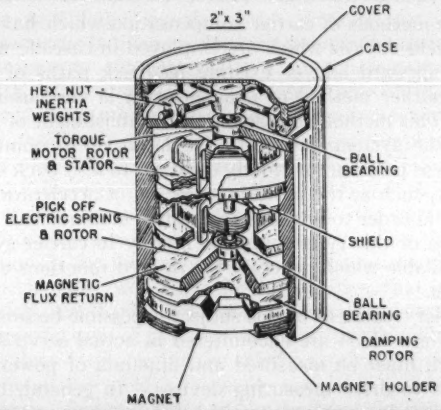

FIG. 12. Mechanical components of compensation device.

(Copied from Ref. 23.)

refined model of the mechanical components (Fig. 12) incorporates the features of variable damping by providing a means of adjusting the distance between the pole faces of the magnet which is included in the eddy-current damper, and of electrically adjustable springs by including a second winding on the pickoff through which a direct current is passed. The value of the direct current determines the force which tends to restore the shaft of the device to its neutral position.

The general method of demodulating, compensating, and remodulating has one inherent advantage, that is, it is independent of carrier-frequency variations. Systems employing electronic demodulators and modulators also have the advantage of making available for carrier compensation all of the types of compensation transfer characteristics

which have long been successfully employed in the non-carrier systems. Electronic demodulators and modulators do have the disadvantage of introducing noise and harmonics into the signal, and although these may be removed to some extent by filtering, the filtering also introduces additional phase lag which may cancel the effectiveness of the over-all response of the system as far as compensation is concerned. Electromechanical systems may be made relatively free of noise, but the compensation characteristics obtained by using them are more limited in number. Considering available knowledge and experience, accurate compensation would probably be more difficult to obtain with electromechanical devices than with all electrical devices.

5.2 Use of Feedback Devices

All of the methods of carrier compensation which have so far been considered yield devices which are employed in cascade with other devices in the forward and/or existing feedback paths of carrier servo systems. Another method of compensation is very useful in certain situations. This method consists of the establishment of new feedback paths from the system output to the input or to points within the system. These paths are normally designed to feed back some function of the output, such as the position, velocity, or acceleration of the output member, in order to improve the system stability and performance. Compensation of this type is applied readily to carrier systems as devices are available which generate the desired functions as modulated-carrier signals.

Only a brief treatment of this subject is possible because of the variety of situations which are encountered in actual servo systems as to outputs which must be measured and amounts of power available at those outputs to drive measuring devices. In general, these outputs will be changing during the operation of the system and will supply information as data frequencies. These data frequencies must be introduced into the system as modulated-carrier signals by the action of the measuring devices. This can easily be accomplished by the use of measuring devices of the electromagnetic-induction type which are excited by the desired carrier voltage and which have their quiescent signal voltage nullified by appropriate electrical balance circuits. The data frequencies will then produce an output from the combination of the measuring device and balance circuit which is a suppressed-carrier modulated signal. Other devices operate on the principle of variations in the reluctance of the magnetic path within the instrument or of variations in the resistance of strain-sensitive wires within the instrument to obtain the modulated-carrier signals.

Feedback devices which have been used in many systems include synchros and rotatable transformers which give signals proportional to the angular position of the output member, induction generators or

tachometers which give signals proportional to the angular velocity of the output member, angular accelerometers which give signals proportional to the angular acceleration of the output member, and potentiometers in appropriate bridge circuits which give signals proportional to output position, either linear or angular. Books edited by James, Nichols, and Phillips (10) and by Blackburn (24) discuss certain of these devices.

5.3 Minimization of Effects of Carrier-Frequency Instability

Notthoff (25), in connection with recent investigations at the Massachusetts Institute of Technology, has shown that, theoretically, pure phase lead compensation which is independent of carrier-frequency variations may be obtained by devices which act directly on the suppressed-carrier modulated signals. Such devices have a constant, positive, phase-*versus*-frequency slope characteristic and a constant, zero slope, amplitude characteristic. These characteristics are not compatible with the minimum-phase-shift structures which are normally employed, but they may be closely approximated over any given range by a physically realizable structure of a minimum complexity of fourth order. The use of an approximation to the desired characteristic limits the usefulness of the device through the introduction of changes in servo resonant frequency and servo peak response as the carrier frequency varies. Notthoff has developed methods for including the carrier-frequency variation in the design procedure. These methods permit the designer to specify any two of the three quantities, that is, carrier-frequency variation, servo peak response, and servo resonant frequency, but with the specification of two of these quantities he will be limited to the range of values available for the third quantity, provided that the two specified quantities yield a physically realizable system.

The specific problem of interest to Notthoff was the provision of lead compensation when the carrier frequency is stabilized only to within plus and minus ten per cent of its nominal value and when the bandwidth of signal frequencies is plus and minus five per cent of the nominal value of the carrier frequency. An investigation of several possible methods of solving this problem resulted in the selection of the following system which is the lowest-order approximation to the theoretical characteristic which gives pure phase lead independent of the carrier frequency.

By plotting the phase and the amplitude of the function

$$G(j\omega) = \frac{\left[1 - \left(\frac{\omega}{\omega_2}\right)^2 + j2\zeta_2\left(\frac{\omega}{\omega_2}\right)\right]\left[1 - \left(\frac{\omega}{\omega_3}\right)^2 + j2\zeta_3\left(\frac{\omega}{\omega_3}\right)\right]}{\left[1 - \left(\frac{\omega}{\omega_1}\right)^2 + j2\zeta_1\left(\frac{\omega}{\omega_1}\right)\right]\left[1 - \left(\frac{\omega}{\omega_4}\right)^2 + j2\zeta_4\left(\frac{\omega}{\omega_4}\right)\right]},$$

where $\omega_1 < \omega_2 < \omega_c < \omega_3 < \omega_4$, and ω_c = carrier frequency, against a logarithmic frequency scale for various values of the ω's and ζ's, a set of

constants may be found which will give a phase characteristic which has an essentially constant positive slope in the region of the carrier frequency plus and minus twenty per cent. Actually this work should be carried out on an arithmetical frequency scale plot but a logarithmic scale is chosen because of the greater ease in the graphical construction and because the over-all accuracy of the method is not greatly affected as the band of frequencies of interest are a small percentage of the carrier frequency. The slope of the phase characteristic may be changed by variation of the constants in order that the function will introduce the desired amount of phase lead for certain data frequencies. The corresponding amplitude characteristic will have a sharp dip in the vicinity of the nominal value of the carrier frequency; however, an analysis by Newton's methods (6) will show that the even part of the amplitude response, which is the part that is effective in the servo system response, is relatively constant for designated data frequencies as the carrier frequency varies over its permissible range. As much as 90° phase shift of designated data frequencies may be obtained with only 2*db* increase in amplitude at those frequencies.

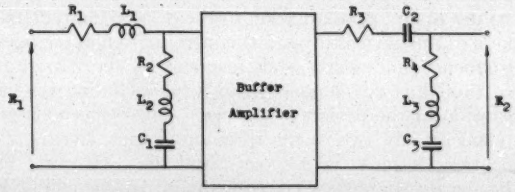

FIG. 13. Notthoff's system of compensation.

In the application of this method of compensation, as the carrier frequency varies the servo system loop gain will change slightly and the carrier frequency will be shifted in phase. These faults may be minimized in certain systems by including appropriate compensation devices in other portions of the systems. For example, in a two-phase-motor servo system the reference voltage for the motor may be shifted in phase with variation in carrier frequency in order to maintain its quadrature relationship with the signal carrier voltage and its magnitude may be varied to compensate for possible changes in loop gain. Notthoff reports very satisfactory application of such a system.

The transfer characteristic which is symbolized by the formula in a previous paragraph may be realized in a relatively simple manner. The first quadratic terms of the numerator and denominator may be obtained by using the equivalent of a non-carrier, quadratic integral network, and the second term may be obtained by using the equivalent of a non-carrier, quadratic lead network. These networks are separated

by a buffer amplifier to prevent interaction. To offset some of the difficulties in the accurate design of networks employing inductances, these networks are constructed with variable inductances and resistances of approximately the correct values and then are adjusted on the basis of laboratory tests to obtain the desired response. The circuit is illustrated in Fig. 13.

6. COMPARISON OF METHODS OF CARRIER COMPENSATION

After discussion of the requirements for carrier-compensation devices and various methods of obtaining carrier-compensation transfer characteristics, the problem must be faced as to which method of compensation should be employed for a particular application. The answer to this problem is not easy to obtain as very little information is available on the comparative efficiency of the various methods. This chapter will attempt to present some of the arguments for and against the various methods. A more basic consideration will be presented also, that is, the evaluation of the response obtained from a carrier servo system. This consideration resolves itself into the consideration of the relative merits of non-carrier and carrier servo systems. In concluding this paper, a summary of the work which has been done in the field of carrier compensation will be given, and suggestions will be made for future work.

6.1 Relative Advantages and Disadvantages of Various Methods of Carrier Compensation

As stated in Section 2.3, there are essentially two types of carrier-compensation devices, namely, those which operate on the carrier signal directly, and those which effectively demodulate the carrier signal, compensate the non-carrier signal, and then remodulate the signal on the carrier. Of the first type of devices this paper has discussed passive electrical networks, whereas of the latter type, electrical and electromechanical methods have been considered. Certain feedback devices which are useful under some conditions have been mentioned also. The various sections of this paper have attempted to evaluate the advantages and disadvantages of the devices considered individually; however, very little has been said so far as to the comparative advantages and disadvantages of the available devices. Such evaluation must now be attempted in order that there may be a logical basis for the choice of a certain device for a particular application.

To the author's knowledge there are no published data available for each of several types of carrier-compensation devices in order that the relative accuracy of those devices in obtaining the desired transfer characteristics may be compared. The nonavailability of these data is unfortunate as in many applications where accurate performance is essential all other considerations in regard to the compensation device are

secondary to the consideration of obtaining accurately the required compensation characteristic. Under the present state of development of compensation devices this limitation assumes lesser importance as only simple carrier lead compensation may be readily obtained by a variety of devices. For the majority of the compensation transfer characteristics a device must be chosen simply because it will give the desired characteristic rather than because it will give the characteristic better than another device.

Secondary considerations in the choice of a particular compensation device, when the possibility for a choice exists, include the ease with which the device can be designed, constructed, and adjusted, the adaptability of the device to existing or proposed equipment, the size, weight, and power requirements of the device, and the cost of the device, not necessarily listed in the order of relative importance. Of course the effect of changes in the components of the device with time or temperature should be included under the consideration of the accuracy of the device in producing the desired transfer characteristic.

Simple carrier lead compensation may be obtained with most of the devices which have been discussed. McDonald (23) gives no data on the accuracy with which his electromechanical devices (see Section 5.1) will give a desired lead transfer characteristic; however, Sobczyk (7) does give an indication of the tolerances required on the components of certain passive electrical networks in order to obtain a specified accuracy in the response (see Section 4.3, for an example). Electrical and electromechanical devices of the demodulation, compensation, and remodulation type do have the inherent advantage in this case, as well as in all other cases, of giving compensation transfer characteristics which are independent of variations in the carrier frequency, but this may not be of too great importance for lead compensation if the carrier frequency is stabilized to five or ten per cent of the desired bandwidth of the lead compensation. Passive electrical R-C networks, such as the parallel-T, bridge-T, and lattice, do appear to be far superior to other methods insofar as ease of design, construction and adjustment, cost, adaptability to existing equipment, and power requirements are concerned. The R-L-C lead network and McDonald's electromechanical device do present the possibiliby of obtaining certain lead characteristics which cannot be obtained with R-C networks.

Simple carrier integral compensation may be obtained only with devices which demodulate the carrier signal and compensate the non-carrier data frequencies. A choice may be made between the use of an all electrical circuit or the attempted use of a modification of the electromechanical compensation device as suggested by McDonald. If the compensation is not too critical, the latter type of device may be more acceptable as it eliminates the use of an electronic modulator. Electronic modulators are actually one of the main drawbacks to the use of

all electrical devices as they normally introduce much distortion and, with proper filtering, much phase lag.

Little possibility exists in choice of the type of device which may be used with greatest ease and accuracy for other types of carrier compensation. Actually, the need of using compensation transfer characteristics which are more complex than simple lead compensation may well cause the designer to build large portions of the servo on a non-carrier basis in order to be able to employ non-carrier compensation methods rather than carrier compensation methods.

In general, when carrier methods are employed, passive electrical networks present the most desirable type of devices, if they can be designed to give the required transfer characteristics. R-C networks are preferable to R-L-C networks as they eliminate the use of inductances which may introduce serious nonlinearities; however, both types of networks can be easily included in the design of the electronic circuits associated with a servo system; they can be synthesized for the required characteristics through the use of the rich knowledge of network design techniques; and unless the networks include an excessively large number of elements of special size or tolerance, they can be built inexpensively. Passive electrical networks can give very dependable response, but perhaps their main limitation is that such response is stable on an absolute frequency scale, hence will change relative to the carrier frequency as the carrier frequency varies. Devices of the demodulation, compensation, and remodulation type are relatively insensitive to changes in carrier frequency, but they are in general so much more complex than passive electrical networks that they may not give as dependable over-all response. Naturally, the increased complexity indicates increased costs, more difficult design problems, and probably increased space requirements.

6.2 Evaluation of Response of Systems Utilizing Carrier Compensation

Given a certain carrier servo system, the question must be considered as to how much the response may be improved by using appropriate carrier-compensation devices. Here again the lack of specific data prohibits a definite answer to the question, but the question may be considered in a more general way such as, when may improvement be expected by using carrier-compensation techniques, and when should portions of the system be designed to employ the more dependable non-carrier compensation techniques. A further consideration reaches the basis of the problem, that is, why and when should carrier servo systems be employed in preference to non-carrier systems.

Available literature (**3, 4, 7, 10, 23**) indicates that extensive use has been made of only simple carrier lead compensation. Such compensation has been employed very effectively in the stabilization of two-phase-motor servo systems and in the improvement of system response.

Carrier integral compensation (23) has been attempted, and perhaps other types of compensation characteristics have been employed, but no basis is available for evaluating the improvement obtained in the response of the systems. As most carrier-compensation transfer characteristics, other than the simple lead compensation characteristic, are relatively difficult to obtain, the employment of non-carrier sections in the servo system for the purpose of utilizing non-carrier compensation techniques would be indicated.

Carrier servo systems do have several inherent advantages as compared to non-carrier systems; therefore, there is a great argument for using them where possible even though system compensation may be relatively difficult. In carrier systems R-C- and transformer-coupled electronic amplifier stages which can be designed and maintained much more easily than direct-coupled stages can be employed. Also, data measurement and transmission devices based on the electromagnetic induction principle can be used to give much smoother and more continuous control than is available from such non-carrier devices as wire-wound potentiometers. Carrier signals usually can be transmitted more readily than non-carrier signals, and, if necessary, various portions of a carrier system may be electrically isolated, except for the carrier signals, by the use of transformers which can also serve the purposes of matching impedance and of providing voltage amplification. The overall result of these characteristics of a carrier system is that there is less phase lag in the system; hence, the possibility of obtaining much faster response from the carrier servo system, with consequently less need for compensation, exists. One situation when a non-carrier system is more suitable than a carrier system is when there is great deal of noise in the system at frequencies only slightly higher than the data frequencies of interest. In such a case a non-carrier system may be provided with appropriate filters with sharp cutoff to attenuate the noise and perhaps reduce the cause of the noise, whereas such is not possible in a carrier system.

6.3 Conclusions

The field of carrier compensation for servomechanisms is only in its infancy, and because of the inherent advantage of carrier servo systems great advances may be expected during the coming years. The most significant contributions to the field, as far as published literature which has been available to the author is concerned, are the reports by Sobczyk (7) and McDonald (23); however, as stated in the introduction to the first chapter of this paper, much of the research in the field of servomechanisms has not been published. No doubt, many applications have been made of various methods of simple carrier compensation and the hope is expressed that these methods may be further studied and the

design data published so that the average designer may benefit from the experience of others.

One of the fields of information which may well be adapted for use in servo compensation is electrical filter theory. Actually, many of the desired types of carrier compensation characteristics are variations of types of filters, so a logical development would be to utilize the wealth of information which is available on filter design. Another possibility for development is further application of generalized feedback theory to servo system design problems in order to obtain systems which require less compensation.

REFERENCES

(1) G. S. BROWN AND D. P. CAMPBELL, "Principles of Servomechanisms," New York, John Wiley & Sons, Inc., 1948.

(2) A. C. HALL, "The Analysis and Synthesis of Linear Servomechanisms," Cambridge, The Technology Press, MIT, 1943.

(3) F. J. LARSEN, "Ultrasonic Trainer Circuits," *Electronics*, Vol. 19, June, 1946, pp. 126–129.

(4) T. P. HEUCHLING, "Tuned L-C Networks for A-C Servos," Electrical Engineering Thesis, Massachusetts Institute of Technology, 1948.

(5) H. T. HARRIS, "The Analysis and Design of Servomechanisms," *Trans. ASME*, Vol. 69, pp. 267–280 (1947).

(6) G. C. NEWTON, "Compensating Networks," Chap. III, Classnotes for Servomechanisms 6.606–6.608, MIT, Cambridge, Mass., 1948.

(7) A. SOBCZYK, "Parallel 'T' Stabilizing Networks for AC Servos," Report 811, Radiation Laboratory, MIT, Cambridge, Mass., 1946; also published with certain revisions as "Stabilization of Carrier Frequency Servomechanisms," JOURNAL FRANKLIN INSTITUTE, Vol. 246, pp. 21–43, 95–121, 187–213 (1948).

(8) W. M. PEACE, "Response of a Network to Suppressed Carrier Input," Engr. Report No. 11, DIC Project 6546, Servomechanisms Laboratory, MIT, Cambridge, Mass., 1948.

(9) E. C. CHERRY, "The Transmission Characteristics of Asymmetric-Sideband Communication Networks," *Jour. Inst. Electrical Engineers*, Part I. Vol. 89, Pt. III, pp. 19–39 (1942), and Part II, *Ibid.*, Vol. 90, Pt. III, pp. 75–88 (1943).

(10) H. M. JAMES, N. B. NICHOLS AND R. S. PHILLIPS, Editors, "Theory of Servomechanisms," New York, McGraw-Hill Book Co., Inc., 1947.

(11) H. LAUER, R. LESNICK AND L. E. MATSON, "Servomechanism Fundamentals," New York, McGraw-Hill Book Co., Inc., 1947.

(12) E. A. GUILLEMIN, "RC-Coupling Networks," Report 43, Radiation Laboratory, MIT, Cambridge, Mass., 1944; E. A. GUILLEMIN, Classnotes for Advanced Network Theory 6.562, MIT, Cambridge, Mass., 1944.

(13) E. A. GUILLEMIN, "Communication Networks," Vol. II, New York, John Wiley & Sons, Inc., 1935.

(14) H. H. SCOTT, "A New Type of Selective Circuit and Some Applications," *Proc. I.R.E.*, Vol. 26, pp. 226–235 (1938).

(15) W. N. TUTTLE, "Bridged-T and Parallel-T Null Circuits for Measurements at Radio Frequencies," *Ibid.*, Vol. 28, pp. 23–29 (1940).

(16) L. STANTON, "Theory and Application of Parallel-T Resistance-Capacitance Frequency Selective Networks," *Ibid.*, Vol. 34, pp. 447–456 (1946).

(17) A. E. HASTING, "Analysis of a Resistance-Capacitance Parallel-T Network and Applications," *Ibid.*, Vol. 34, pp. 126–129 (1946).

(18) A. WOLF, "Note on a Parallel-T Resistance Capacitance Network," *Ibid.*, Vol. 34, p. 659 (1946).

(19) G. J. THIESSEN, "R-C Filter Circuits," *Jour. Acoustical Soc. America*, Vol. 16, pp. 275–279 (1945).
(20) M. P. GIVENS AND J. S. SABY, "A Simplified Analysis of a Parallel-T Null Network," *Rev. Scientific Instruments*, Vol. 18, pp. 342–346 (1947).
(21) F. E. TERMAN, "Radio Engineers Handbook," New York, McGraw-Hill Book Co., Inc., 1943.
(22) H. W. BODE, "Network Analysis and Feedback Amplifier Design," New York, D. Van Nostrand and Co., 1945.
(23) D. MCDONALD, "Improvement in the Characteristics of A-C Lead Networks for Servomechanisms," Report No. UMM 25, Aeronautical Research Center, Engineering Research Institute, University of Michigan, Ypsilanti, Mich., October, 1948; Revision B, July, 1949. Also available as: DONALD MCDONALD, "Electromechanical Lead Networks for AC Servomechanisms," *Rev. Scientific Instruments*, Vol. 20, November, 1949, p. 775.
(24) J. F. BLACKBURN, Editor, "Components Handbook," New York, McGraw-Hill Book Co., Inc., 1949.
(25) A. P. NOTTHOFF, JR., "Phase Lead for A-C Servo Systems with Unregulated Carrier," Electrical Engineering Thesis, Massachusetts Institute of Technology, Cambridge, Mass., 1949.

The following references are not mentioned in the text of this paper; however, they contain very helpful information on the compensation of servomechanisms, especially non-carrier systems.

(26) S. W. HEWALD, "Considerations in Servomechanism Design," *Trans. AIEE*, Vol. 63, pp. 871–879 (1944).
(27) J. G. CLARKE, "Differentiating and Integrating Circuits," *Electronics*, Vol. 17, November 1944, pp. 138–144.
(28) R. E. GRAHAM, "Linear Servo Theory," *Bell System Tech. Jour.*, Vol. 25, pp. 616–651 (1946).
(29) A. L. WHITLEY, "Theory of Servo Systems with Particular Reference to Stabilization," *Jour. Inst. Electrical Engineers*, Vol. 93, Pt. II, pp. 353–372 (1946).
(30) H. CHESTNUT, "Obtaining Attenuation-Frequency Characteristics for Servomechanisms," *Genetal Electric Rev.*, Vol. 50, December, 1947, pp. 38–44.
(31) G. J. SCHWARTZ, "The Application of Lead Networks and Sinusoidal Analysis to Automatic Control Systems," *Trans. AIEE*, Vol. 66. pp, 69–77 (1947).

SOME NOTES ON THE ANALYSIS OF REDUNDANT SYSTEMS BY MEANS OF THE CONCEPTION OF CONSERVATION OF ENERGY

BY

T. M. CHARLTON [1]

SYNOPSIS

The principles of the analysis of conservative redundant systems capable of deformation are considered. It is well known that reference both to the conditions for equilibrium and geometrical compatibility is necessary to enable a unique solution to be obtained in a particular case. The setting up of one or other of these conditions in a general case using the concept of energy is discussed with reference to the law of conservation of energy. The use of "complementary energy" for the fulfillment of the conditions for geometrical compatibility is demonstrated.

The direct application of the conditions of equilibrium and geometrical compatibility is considered in an appendix.

INTRODUCTION

The classical methods of analyzing conservative redundant systems are well known and include the principle of stationary potential energy (from which the principle of virtual work can be derived) and Castigliano's principle of least work. The former is the most fundamental of the energy methods; it is quite general and depends upon the assumption of geometrical compatibility since it is, in fact, a systematic means of applying the conditions for equilibrium.

The latter is not strictly fundamental; it ensures geometrical compatibility, having applied the conditions for equilibrium, in the case of certain kinds of system only, the most common example of which is the linear elastic system. The comparatively little known principle of stationary complementary (fictitious) energy due to Engesser ensures geometrical compatibility in the general case subject to certain limitations, having assumed equilibrium initially. This principle is not fundamental either; Castigliano's principle is really a particular case of it, since the strain energy in terms of forces, and the complementary energy expression in the same terms are identical for a linear elastic system. The main limitation of the complementary energy principle lies in the fact that conservation of complementary energy is its basis.

Since energy can be considered as a mathematical conception in analysis, all energy methods are, in effect, one ready method of setting up either the conditions for equilibrium or the conditions for geometrical compatibility, having applied the other for simplification of the energy expression. It is well known that in order to analyse an "indetermi-

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nate" structure, appeal must be made to both the conditions of equilibrium and geometrical compatibility.

The purpose of this paper is to indicate a direct approach to the energy methods of analysis via the conception of the conservation of energy.

No practical advantages are claimed for the proposed approach in view of the excellence of relaxation methods for obtaining solutions of the kind required in engineering practice. It is contended, however, that the methods contribute to a better understanding of fundamental principles.

THE ANALYSIS OF ELASTIC SYSTEMS WITH THE AID OF CONSERVATION OF ENERGY—THE PRINCIPLE OF LEAST WORK

It will be assumed, for the moment, that the object of the investigation is to determine the effect of external forces only upon the system, that is, self straining is absent.

The strain energy induced in a conservative elastic system (linear or otherwise) can be expressed either in terms of displacements by referring to the conditions for geometrical compatibility or alternatively in terms of internal and external forces by referring to the equilibrium conditions. The former procedure is fundamental since $\delta W = \delta U = \sum F \cdot \delta y$ by the law of conservation of energy, where W is the external work done; U is the strain energy; F is an external force and y is an external deflection. Thus $\delta(U - W) = 0$ and:

$$\frac{\partial W}{\partial y} = \frac{\partial U}{\partial y} = F. \tag{1}$$

For a linear elastic system, if the internal and external displacements are represented by e's and y's, respectively, and if there are R of the former remaining after application of the compatibility conditions, and n external forces:

$$U = U(e_1, e_2, \cdots, e_R; y_1, y_2, \cdots, y_n) \tag{2}$$

and

$$W = W(y_1, y_2, \cdots y_n). \tag{3}$$

Applying the law of conservation of energy:

$$U(e_1, e_2, \cdots, e_R; y_1, y_2, \cdots, y_n) - W(y_1, y_2, \cdots y_n) = 0 \tag{4}$$

since the variables in Eq. 4 are independent, partial differentiation with respect to each, in turn gives:

$$\frac{\partial U}{\partial y_i} - \frac{\partial W}{\partial y_i} = 0 \quad (i = 1, 2, \cdots, n)$$

and (5)

$$\frac{\partial U}{\partial e_j} - \frac{\partial W}{\partial e_j} = 0 \quad (j = 1, 2, \cdots, R).$$

These relationships will yield sufficient equations to provide a unique solution in a particular case and are similar to those which would have resulted, had the principle of stationary potential energy been applied. The equations will not, generally, be ill-conditioned for a linear system.

If a fixed system of external displacements is applied to a system, by the previous reasoning, Eqs. 5 reduce to:

$$\frac{\partial U}{\partial e_j} = \frac{\partial W}{\partial e_j} = 0. \tag{6}$$

Since the internal displacements e determine the deformed shape of the system under any conditions, it is clear from both Eqs. 5 and 6 that they will always be such as to make the strain energy stationary (or a minimum if U is a positive definite function as in the case of a linear elastic system) with respect to them. It is this principle which is the fundamental principle of least work in the author's opinion.

It will be observed that having applied the conditions of geometrical compatibility, the variables having dimensions of length need not necessarily be retained. Generalized co-ordinates q can be employed, or co-ordinates having dimensions of force. Thus, in general Eqs. 5 become:

$$\frac{\partial U}{\partial q_i} - \frac{\partial W}{\partial q_i} = 0 \quad (i = 1, 2, \cdots, Q), \tag{7}$$

where $Q = (R + n)$.

The application of this procedure is not confined to systems which obey Hooke's Law; on the contrary, it is of general application to conservative systems.

If, on the other hand, it is desired to determine the conditions for compatibility using the energy concept, it is necessary to work in terms of forces and appeal to equilibrium conditions initially to eliminate as many unknowns as possible. In the case of linear elastic systems, the strain energy may be employed, but in the general case the complementary energy is necessary.

Confining the discussion to linear elastic systems for the moment, it is well known that in this case $\delta W = \delta U = \sum y \cdot \delta F$ (although strictly $\sum y \delta F = \delta C$, C being the complementary energy, represented by the area above the load-deflection curve). Therefore:

$$\frac{\partial W}{\partial F} = \frac{\partial U}{\partial F} = y. \tag{8}$$

If the system contains r redundant members the forces in which are T_1, T_2, $\cdots$, T_r and the loading consists of n external forces F then:

$$U = U(T_1, T_2, \cdots, T_r; F_1, F_2, \cdots, F_n) \tag{9}$$

and the external work done is given by:

$$W = \tfrac{1}{2} \sum_{i=1}^{i=n} F_i \cdot y_i = \tfrac{1}{2} \sum_{i=1}^{i=n} \sum_{j=1}^{j=n} F_i \cdot F_j \cdot k_{ij}, \tag{10}$$

where $k_{ij} = k_{ji}$ is an external constant flexibility (deflection) coefficient of the system.

By the law of conservation of energy:

$$U(T_1, T_2, \cdots, T_r; F_1, F_2, \cdots, F_n) - \tfrac{1}{2} \sum_{i=1}^{i=n} \sum_{j=1}^{j=n} F_i \cdot F_j \cdot k_{ij} = 0 \tag{11}$$

and differentiating[2] with respect to each of the independent variables in turn gives:

$$\frac{\partial U}{\partial F_i} = \frac{\partial W}{\partial F_i} = y_i \quad (i = 1, 2, \cdots, n)$$

and (12)

$$\frac{\partial U}{\partial T_j} = \frac{\partial W}{\partial T_j} = 0 \quad (j = 1, 2, \cdots, r).$$

The second group of Eqs. 12 provides the necessary additional (compatibility) conditions for a unique solution and represents Castigliano's least work principle. These equations will, however, be ill-conditioned in all probability.

In general, however, Eq. 8 is incorrect although there are particular cases other than the linear elastic system for which the equality is valid. Since the complementary energy of a system $C_E = \sum \int y dF = C_I = \sum \int e dT$ within a certain range, where C_E represents the external complementary energy and C_I represents the internal complementary energy, it is clear that the partial differential coefficient of complementary energy with respect to force gives deflection.

The reason for the validity of "conservation" of complementary energy stated above, lies in the validity of the condition for the total potential energy to be stationary in terms of finite displacements, or the principle of virtual work for finite displacements. That is, in the validity of:

$$\Delta V = + \sum_{j=1}^{j=N} T_j \cdot e_j - \sum_{i=1}^{i=n} F_i \cdot y_i = 0, \tag{13}$$

where V represents potential energy and N the total number of members within the system. The displacements $e_j (j = 1, 2, \cdots, N)$ and y_i $(i = 1, 2, \cdots, n)$ are geometrically compatible arbitrary finite dis-

[2] This is not fundamental since, in general, strain energy can only be varied by a variation of displacements.

placements in general but for the present investigation they will be assumed to be the displacements actually produced by the respective forces. Finite displacements can be considered, provided that they are within a range within which the principle of superposition applies to the displacements, or displaced form of the system only and that the working range is within the same range. In actual practice the principle of superposition can be applied to displacements with good approximation, generally, provided that only second order variations in the over-all geometrical form of the system are involved.

Now by the law of conservation of energy:

$$+\sum_{j=1}^{j=N}\int_0^{e_j} T_j \cdot de_j - \sum_{i=1}^{i=n}\int_0^{y_i} F_i \cdot dy_i = 0. \tag{14}$$

Subtracting the left hand side of Eq. 14 from that of Eq. 13 gives:

$$+\sum_{j=1}^{j=N}\int_0^{T_j} e_j \cdot dT_j - \sum_{i=1}^{i=n}\int_0^{F_i} y_i \cdot dF_i = 0 \tag{15}$$

or

$$C_I - C_E = 0$$

and

$$\delta(C_I - C_E) = 0. \tag{16}$$

Having established the "law of conservation of complementary energy," this may be employed as a means for obtaining the compatibility conditions for a system in a similar manner to that in which the law of conservation of energy is employed above as a means of obtaining the equilibrium conditions.

Thus, the internal complementary energy of a system can be expressed as:

$$C_I = C_I(T_1, T_2, \cdots, T_r; F_1, F_2, \cdots, F_n) \tag{17}$$

having applied the equilibrium conditions; and the external complementary energy can be expressed as:

$$C_E = C_E(F_1, F_2, \cdots, F_n). \tag{18}$$

It will be observed that these two quantities are respectively analogous to strain energy and external work done. Now, within the prescribed limits:

$$C_I(T_1, T_2, \cdots, T_r; F_1, F_2, \cdots, F_n) - C_E(F_1, F_2, \cdots, F_n) = 0 \tag{19}$$

as demonstrated above. Differentiating partially with respect to each variable in turn gives the following equations:

$$\frac{\partial C_I}{\partial F_i} - \frac{\partial C_E}{\partial F_i} = 0 \quad (i = 1, 2, \cdots, n) \tag{20i}$$

and

$$\frac{\partial C_I}{\partial T_j} - \frac{\partial C_E}{\partial T_j} = 0 \quad (j = 1, 2, \cdots, r). \tag{20ii}$$

The Eqs. 20ii are sufficient to enable the analysis of a system to be completed and indicate that the complementary energy is stationary with respect to internal forces. This is Engesser's principle which was referred to previously. Its physical interpretation is obscure, as is that of Castigliano's principle of least work.

It will have been noticed that the equations given herein bear resemblance to those obtained for pin jointed structures but this is for reasons of simplicity only.

SELF STRAINING

If U_o is the energy stored in a conservative system due to self straining and U_W is the strain energy stored as a result of the work done by external forces, the total strain energy U is given by:

$$U = U_o + U_W. \tag{21}$$

If the system is considered as pin jointed for the sake of simplicity, then assuming that the self straining is due to the initial lack of fit of each of the redundant members to the extent of $\lambda_i (i = 1, 2, \cdots, r)$, the fundamental expression for the strain energy will be:

$$U = U(\lambda_1, \lambda_2, \cdots, \lambda_r; e_1, e_2, \cdots, e_R; y_1, y_2, \cdots, y_n); \tag{22}$$

and that for the external work done will be:

$$W = W(\lambda_1, \lambda_2, \cdots, \lambda_r; y_1, y_2, \cdots, y_n). \tag{23}$$

This includes for the external work done in constructing the frame, which is equal to the energy of self-straining.

Equating expressions (22) and (23) by the law of conservation of energy and differentiating with respect to each unknown in turn (the compatibility conditions having been applied initially) the following equations will be obtained:

$$\begin{aligned} &\frac{\partial U}{\partial y_i} - \frac{\partial W}{\partial y_i} = 0 \quad (i = 1, 2, \cdots, n) \\ &\frac{\partial U}{\partial e_j} - \frac{\partial W}{\partial e_j} = 0 \quad (j = 1, 2, \cdots, R). \end{aligned} \tag{24}$$

Since the λ's are constants, these equations are similar to those of (8) and those resulting from application of the principle of stationary potential energy.

When $U_W = 0$, then $U = U_0 = U(\lambda_1, \lambda_2, \cdots, \lambda_r; e_1, e_2, \cdots, e_R)$ and

$W = W(\lambda_1, \lambda_2, \cdots, \lambda_r)$ and:

$$\frac{\partial U}{\partial e_i} = \frac{\partial W}{\partial e_i} = 0 \quad (i = 1, 2, \cdots, R) \tag{25}$$

since the λ's are constants as before.

If it is necessary to work in terms of forces to determine the compatibility conditions, complementary energy is the fundamental quantity (strain energy in the case of linear elastic systems only). Thus, as above:

$$C_I = C_I(T_1, T_2, \cdots, T_r; F_1, F_2, \cdots, F_n) \tag{26}$$

having applied the equilibrium conditions; and:

$$C_E = C_E(T_{1\cdot 0}, T_{2\cdot 0}, \cdots, T_{r\cdot 0}; F_1, F_2, \cdots, F_n), \tag{27}$$

where $T_{1\cdot 0}$ represents the force due to self-straining only in the ith redundant member.

Conservation of complementary energy gives:

$$C_I(T_1, T_2, \cdots, T_r; F_1, F_2, \cdots, F_n) - C_E(T_{1\cdot 0}, T_{2\cdot 0}, \cdots, T_{r\cdot 0}; F_1, F_2, \cdots, F_n) = 0 \tag{28}$$

and as before

$$\frac{\partial C_I}{\partial F_i} - \frac{\partial C_E}{\partial F_i} = 0 \quad (i = 1, 2, \cdots, n), \tag{29}$$

while

$$\frac{\partial C_I}{\partial T_j} - \frac{\partial C_E}{\partial T_{j\cdot 0}} = 0 \quad (j = 1, 2, \cdots, r), \tag{30}$$

since $T_j = T_{j\cdot 0} + T_j'$, where T_j' is due to the external forces, also $\frac{\partial C_E}{\partial T_{j\cdot 0}} = \lambda_j$ because:

$$\delta C_E = \sum y_i \cdot \delta F_i + \sum \lambda_j \cdot \delta T_{j\cdot 0}.$$

Equations 30 are sufficient to enable the compatibility conditions to be determined.

CONCLUSION

It is considered that the whole conception of the energy methods applicable to structural theory is better appreciated if initial reference is made to the law of conservation of energy in fundamental terms. This course eliminates the necessity for reference to the various disjointed theorems named after their respective originators and mainly applicable to the theory of linear elastic structures only. Furthermore, the theory of linear elastic structures often causes the various distinct principles of mechanics to be confused.

The principle of stationary potential energy is fundamental but, unfortunately, it is usually neglected; its derivation is well known.

The methods given above are of general application to conservative statical systems and are not limited to those which obey Hooke's Law.

APPENDIX

Conditions of Equilibrium and Geometrical Compatability Applied Directly to the Solution of Redundant Systems

For the sake of simplicity and generality, the case of a pin-jointed linear elastic framework, on rigid supports self-strained due to two redundancies, will be considered. Suppose that the initial lack of fit of the two redundant members are λ_1 and λ_2, respectively, and that the axial deformation per unit axial force for the members are k_1 and k_2, and let both of them be of value k for simplicity. In order to fix these members in position initially it will be assumed that one end of each is located first of all and then to locate the opposite ends, equal and opposite forces are applied simultaneously to each of the two members and the appropriate points on the structure, respectively. Let the ultimate values of these forces be $T_{1\cdot0}$ and $T_{2\cdot0}$, respectively, being sufficient to permit final connection.

Thus, if k_{11}, k_{22} and $k_{12} = k_{21}$ are the flexibility or deflection coefficients for the simply stiff structure at the points of connection of the redundant members, referred to the deformations in the directions of the members, then:

$$\lambda_1 = T_{1\cdot0}\cdot(k + k_{11}) + T_{2\cdot0}\cdot k_{12} = \lambda_{m\cdot1} + \lambda_{f\cdot1}$$

say (i)

$$\lambda_2 = T_{1\cdot0}\cdot k_{21} + T_{2\cdot0}\cdot(k + k_{22}) = \lambda_{m\cdot2} + \lambda_{f\cdot2},$$

where: $\lambda_{m\cdot1} = T_{1\cdot0}\cdot k$, the deformation of member 1;

$\lambda_{m\cdot2} = T_{2\cdot0}\cdot k$, the deformation of member 2;

$\lambda_{f\cdot1} = (T_{1\cdot0}\cdot k_{11} + T_{2\cdot0}\cdot k_{12})$, the deformation of the simply stiff frame along line 1, due to $T_{1\cdot0}$ and $T_{2\cdot0}$.

$\lambda_{f\cdot2} = (T_{1\cdot0}\cdot k_{21} + T_{2\cdot0}\cdot k_{22})$, the deformation of the simply stiff frame along line 2, due to $T_{1\cdot0}$ and $T_{2\cdot0}$.

The strain energy due to self-straining is clearly given by:

$$\begin{aligned} U_o &= \tfrac{1}{2}\{T_{1\cdot0}\cdot(\lambda_{m\cdot1} + \lambda_{f\cdot1}) + T_{2\cdot0}\cdot(\lambda_{m\cdot2} + \lambda_{f\cdot2})\} \\ &= \tfrac{1}{2}(T_{1\cdot0}\cdot\lambda_1 + T_{2\cdot0}\cdot\lambda_2) \end{aligned} \quad \text{(ii)}$$

When external forces are applied to the structure such that forces T_1' and T_2' are induced in the redundant members in addition to $T_{1\cdot0}$

and $T_{2\cdot 0}$ due to self-straining, Eqs. i become:

$$\lambda_1 = (T_{1\cdot 0} + T_1')\cdot(k + k_{11}) + (T_{2\cdot 0} + T_2')\cdot k_{12} + \sum_{i=1}^{i=n} F_i\cdot k_{1i}' = T_1\cdot(k + k_{11}) + T_2\cdot k_{12} + \sum_{i=1}^{i=n} F_i\cdot k_{1i}'$$

$$\lambda_2 = (T_{1\cdot 0} + T_1')\cdot k_{21} + (T_{2\cdot 0} + T_2')\cdot(k + k_{22}) + \sum_{i=1}^{i=n} F_i\cdot k_{2i}' = T_1\cdot k_{21} + T_2\cdot(k + k_{22}) + \sum_{i=1}^{i=n} F_i\cdot k_{2i}', \qquad \text{(iii)}$$

where $T_1 = (T_{1\cdot 0} + T_1')$, $T_2 = (T_{2\cdot 0} + T_2')$ and k' is an external deflection coefficient for the simply stiff system. These equations, together with the conditions of equilibrium, are sufficient to enable a unique solution to be obtained. The method is clearly capable of generalization to any degree required, but before solution of the equations can be attempted, the coefficients k must be calculated for the redundant members and the statically determinate system.

The strain energy of the self-strained system with external loading is obtained by adding to expression (ii) the strain energy due to the external forces when there is no self-straining.

BIBLIOGRAPHY

(1) J. CLERK MAXWELL, "On the Calculation of the Equilibrium and Stiffness of Frames," *Phil. Mag.*, 4th Series, Vol. 27, No. 182, April, 1864, p. 294.

(2) R. V. SOUTHWELL, "On Castigliano's Theorem of Least Work and the Principle of St. Venant," *Phil. Mag.*, Vol. 45, 7th Series, pp. 193–212 (1923).

(3) T. POSCHL, "Über die Minimalprinzipe der Elastizitätstheorie," *Der Bauingeniuer*, 1st May 1936, pp. 160–164.

(4) E. H. SALMON, "Materials and Structures," Vol. II, London, Longmans and Green, 1938.

(5) D. WILLIAMS, "Relations between the Energy Theorems Applicable in Structural Theory," *Phil. Mag.*, Vol. 60, 7th Series, pp. 617–635 (1938).

(6) R. V. SOUTHWELL, "An Introduction to the Theory of Elasticity," Oxford University Press, 1941.

(7) I. S. SOKOLNIKOFF, "Mathematical Theory of Elasticity," New York, McGraw Hill Book Co., 1946.

(8) H. L. COX, Discussion of paper "Stresses by Analysis and Experiment," by A. J. S. Pippard, Proc. I. Mech. E. 1946, Vol. 155, War Emergency Issue No. 13, p. 31.

(9) M. Z. KRZYWOBLOCKI, "On the So-Called Principle of Least Work Methods, I and II." JOUR. FRANKLIN INSTITUTE, Vol. 243, No. 3, March 1947, pp. 187–199 and Vol. 244, No. 6, December 1947, pp. 465–569.

(10) T. M. CHARLTON, "The Analysis of Elastic Stability," *Civil Engineering and Public Works Review*, Vol. 44, Nos. 516 and 517, pp. 315–317 and 385–387, June and July, 1949.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

SIMPLIFIED VISCOSITY TEST FOR CELLULOSE

An improved method for determining the quality of cellulose has been devised by H. F. Launer and William K. Wilson of the National Bureau of Standards.[1] The new method is a simplification of the cuprammonium viscosity determination, which is widely used as an aid in determining the effect of light, heat, and chemical reagents on the cellulose in paper and textiles. Advantages of the Bureau's procedure are ease of preparation of the cuprammonium reagent and a reduction in sources of error during the reaction.

The cuprammonium test is one of the most important in cellulose chemistry. It involves the measurement of the time required for a definite quantity of a solution of cellulose in cuprammonium hydroxide to flow from a calibrated viscometer. This time of flow, usually measured at 20°C. on a solution containing 0.5 per cent cellulose, is related to the viscosity of the solution and therefore to the quality of the cellulose.

Cuprammonium is usually prepared by the air-oxidation of metallic copper in ammonium hydroxide. This is a laborious, time-consuming process and is further complicated by the formation of ammonium nitrite, which lessens the usefulness of the solution. As the reagent is unstable at room temperature, it must be stored at 0°C. Another disadvantage in the cuprammonium test as ordinarily carried out is the necessity of using special apparatus to exclude air during the solution of the cellulose since cellulose is particularly sensitive to oxidation.

The new procedure removes the difficulties in both stages of the process. The cuprammonium reagent is first prepared by dissolving solid cupric hydroxide in ammonium hydroxide in an ice bath. This easy method of preparation makes storage unnecessary. Cellulose is then dissolved in the reagent, with the addition of cuprous chloride and copper wire to prevent air oxidation. Rubber-stoppered Erlenmeyer flasks are convenient for this purpose and mechanical agitation facilitates the process. The time of flow of the resulting solution is measured in a viscometer enclosed in a glass jacket, through which water is pumped from a constant-temperature bath.

This procedure has been found to save both time and labor. The results compare favorably with those obtained by the older method.

* Communicated by the Director.

[1] For further technical details, see "Preparing Cuprammonium Solvent and Cellulose Solutions," by H. F. Launer and W. K. Wilson, *Analytical Chemistry*, Vol. 22, p. 455 (1950). Dr. Launer is now associated with the Western Research Laboratory, U. S. Department of Agriculture, Albany, Calif.

VIBRATORY BALL MILL FOR MORE EFFICIENT GRINDING

A vibratory ball mill recently developed at the National Bureau of Standards rapidly and completely grinds cotton into particles a few microns in length with little or no oxidative change or contamination of the cellulose. Grinding takes place as 3700 steel balls, colliding over 100,000 times per second, pulverize the material caught between them. As the new mill [1] produces a more uniform powder and is more efficient and easier to operate than previous devices of this kind, it is expected to find extensive application in the grinding and blending of a variety of substances such as pigments, ceramic materials, metal powders, resins, and plant and animal tissues.

The mill was constructed by Florence Forziati, W. K. Stone, J. W. Rowen, and W. D. Appel of the Bureau staff in connection with a program sponsored by the U. S. Department of Agriculture on the chemical deterioration of cotton during processing and use. To learn more about the chemical changes which occur when cotton deteriorates, the Bureau undertook to apply the techniques of infrared spectrophotometry to cotton cellulose. As the spectra obtained from fibers laid side by side were not satisfactory, it was decided to reduce the fibers to a powder and measure the spectral absorption of the powder in a mat or in suspension in a suitable liquid. Several well-known methods for grinding materials were therefore tried, such as an ultrasonic generator and a Wiley mill in which the material is cut between fixed and moving blades. However, the products thus obtained contained much coarse matter. In an effort to provide a method with would reduce the entire sample to a finely divided state without contamination and with a minimum of chemical change, the vibratory ball mill was developed.

In the Bureau's mill, a cylindrical jar suspended by leaf springs and containing steel balls in contact with the material to be ground is swung through a circular path.[2] This excites the steel balls, causing the numerous collisions that grind the material in the jar. At the same time the jar rotates slowly about its axis. In this way, the material being ground is prevented from settling to the bottom of the jar, and uniform grinding is obtained.

Motion of the jar is produced by means of a horizontal shaft connected at one end, through a flexible coupling, to a 0.25-hp. motor and at the other end to an eccentric weight. The weight, rotating at a speed of 1800 rpm., drives the weight housing in a circular path that is con-

[1] For further details, see "Cotton powder for infrared transmission measurements," by Florence H. Forziati, Walter D. Stone, John W. Rowen, and Wm. D. Appel, *J. Research NBS*, Vol. 45 (1950) RP2116. Working drawings of the mill may be obtained upon request from the Textiles Section, National Bureau of Standards, Wash., D. C.

[2] A mill using this general principle was previously constructed by Dr. P. H. Hermans, Utrecht, Netherlands. See P. H. Hermans and A. Weidinger, *J. Am. Chem. Soc.*, Vol. 68, p. 2547 (1946).

fined within certain limits by the flexural resistance of the springs. The jar containing the sample is held in a cylindrical holder, which is attached to the weight housing by means of a greased slip ring. A small difference in diameter between the flange of the jar holder and the slip ring groove permits the holder with the clamped-in jar to be rolled around the groove circumference by centrifugal force. This rolling or rotation of the holder and jar, which takes place at a rate of about 4 rpm. in a direction opposite to that of the rotating weight, continually redistributes the material being ground.

In experiments at the National Bureau of Standards, the mill reduced 5 g. of cotton in 30 min. to particles 10 microns or less in major dimension. X-ray diffraction measurements showed that the cellulose was converted almost completely to the amorphous form in this time. Except for the decrease in the degree of polymerization which resulted from the cutting of the cellulose chains, there was no degradation of the cotton. Nor was the ash content of the cotton perceptibly increased by the grinding process.

NEW pH STANDARDS

Four new pH standards, intended to provide fixed points at the upper and lower ends of a standard pH scale, have been selected by chemists of the National Bureau of Standards. The new solutions extend the accuracy of standard scales made to conform with the three middle-range standards[1] which the Bureau now distributes to the chemical industry.

Measurements of the degree of acidity and alkalinity play an important part in the research laboratory and in the regulation of such modern industrial processes as food preparation, sugar refining and the manufacture of paper, textiles and dyes. These measurements are expressed in terms of hydrogen-ion concentration, or pH, on a scale ranging from a pH of almost zero (very high acidity) to a value near 14 (very high alkalinity). Although exact measurements of acidity can be made with instruments now in use, there has been a lack of uniformity among the results obtained because of the variety of definitions of the pH unit popular among chemists and the consequent difference in methods of calibrating the instruments.

The National Bureau of Standards has long recognized the need of a universally accepted scale of acidity, analogous to the International Temperature Scale. To encourage the adoption of such a scale, the Bureau has, for several years, distributed three standard samples of known pH in the United States and abroad. These samples fix the scale at pH 4, 7, and 9, and the pH of intermediate points is obtained by

[1] For further technical details, see "Standardization of the pH scale," *NBS Technical News Bulletin*, Vol. 31, p. 138 (1947).

interpolation. Unfortunately, this scale is not uniform at its ends, and additional standards are needed to reduce the error of measurement at low and high pH.

In a recent study, R. G. Bates, Gladys G. Pinching and E. R. Smith of the Bureau's physical chemistry laboratory determined the extent of inaccuracy at the ends of the practical pH scale. They concluded that an instrument calibrated with the present solutions of intermediate acidity will usually yield results that are low by 0.02 to 0.05 unit or more above pH 11. Below pH 2.5, errors as great as 0.03 unit, either positive or negative, were found. In general, solutions of strong acids and bases display larger errors than buffered solutions.

In the course of the work 41 solutions of possible value were examined. Out of these, the following four standards were selected:

1. A mixture of sodium bicarbonate and sodium carbonate, both at 0.025 molar concentration, with a pH of about 10 at room temperature.
2. A 0.01 molar solution of trisodium phosphate, pH about 11.7.
3. A solution of potassium hydrogen tartrate saturated at room temperature, pH about 3.6.
4. A 0.01 molar solution of potassium tetroxalate, pH about 2.1.

Although the new standards lengthen the accurate portion of the scale in both directions, the measured pH of highly acid and highly alkaline solutions must still be assigned an uncertainty of at least $\pm$ 0.03 unit.

The Bureau plans to add the potassium hydrogen tartrate and the potassium tetroxalate to the Standard Samples it issues when an adequate supply of the purified materials has been obtained, tested and bottled for distribution. An announcement of their availability will be made in the *Technical News Bulletin*.

ELECTRODYNAMIC AMMETER FOR VERY-HIGH FREQUENCIES

In establishing standards for electrical circuits in the very-high-frequency region now so widely used by radio and television services it is important to extend the direct measurement techniques used at lower frequencies as far as possible. Up to 300 megacycles per second the current flowing in a circuit whose physical dimensions are small with respect to wavelength is essentially a uniform quantity, and the electrical characteristics of small circuit elements may be determined directly in terms of voltage and current. This fact makes possible the establishment of a standard electrodynamic ammeter for the VHF range.

Such an electrodynamic ammeter design, employing a short-circuited ring coupled to a coaxial transmission line, has been the subject of a theoretical and experimental study by Max Solow of the National

Bureau of Standards. His work extends a previous study by Turner and Michel at Yale University. Basically the method depends on a torque measurement on a conducting ring immersed in a field which does not change with frequency. This technique provides an absolute, broad-band measurement of high-frequency current, but several factors are critical in any actual design.

For minimum distributed capacitance and uniform current the short-circuited ring must be only a single turn, and the ring diameter must be small with respect to wavelength. For accurate inductance calculations the ring conductor should have a small cross section, but resistance then limits the current. A ring 1 cm. in diameter of No. 20 copper wire is a practical size. When the ring current is small, the torque is also small, and the ring must be suspended on a delicate quartz fiber for accurate torque measurements. The coaxial line, acting as the primary current-carrying element for the electrodynamic ammeter, has several advantages over other forms of conductor. Its electromagnetic field can be calculated in a straightforward manner, and the line may be readily modified for calibration work with different types of radio-frequency ammeters.

Calibration of the electrodynamic ammeter may be accomplished directly and absolutely. A section of the coaxial transmission line, one wavelength long at 300 megacycles, is arranged with short-circuited ends to form a resonant cavity, and the torque ring is placed midway along the section. A known value of 300 megacycle power is fed into an input loop at one end of the cavity. Under these conditions the torque ring will be at a current maximum and a voltage minimum, and the measured torque on the ring will be due almost entirely to the magnetic component of the cavity field. The measurement is then repeated at 150 megacycles where the current and voltage relations are reversed and the torque is due only to the electric component. One further measurement is needed for absolute calibration of the ammeter. The cavity resonance frequency is measured at both 300 and 150 megacycles with and without the torque ring in place. The resulting changes in frequency are then a measure of the field discontinuity introduced by the presence of the ring. After the torque and discontinuity measurements are completed the instrument will be ready for use as a standard to calibrate other ammeters at very high frequencies.

THE FRANKLIN INSTITUTE

MEDAL DAY PROCEEDINGS

The annual reception, dinner, and presentation of awards, known as "Medal Day," took place at The Franklin Institute on Wednesday, October 18, 1950. The reception began at 6:30 P.M. and dinner was served at 7:30 P.M. Mr. Richard T. Nalle, President of the Institute, presided at the dinner and presented the subsequent awards.

Reception

The usual reception preceded the dinner and was attended by 260 persons. Immediately following the reception and just before dinner was served in Franklin Hall, Mr. Nalle requested the guests to face the flag above the Franklin statue while Mr. Guy Marriner, director of music at The Institute, played the National Anthem.

Remarks of the President

Mr. Nalle: "Tonight we recognize a group of individuals who, by dedicating their talents and energies to the investigation of scientific problems, have furthered the usefulness of science in our daily lives.

"Benjamin Franklin, whom this Memorial honors and for whom the Institute is named, made continuous and successful efforts to spread the knowledge of scientific things and to show their application to every-day life, as well as to solve the problems and mysteries that confronted him.

"That great American was devotedly interested in developing the minds of those with scientific leanings and he was constantly striving to encourage and reward their achievements. He would most certainly approve of the scientists gathered here now.

"Therefore, ladies and gentlemen, I propose a toast on this 1950 Medal Day to the living memory of Benjamin Franklin."

* * *

Mr. Nalle (*continued*): "Honored guests, members of The Franklin Institute, and friends:

"It is with deep pleasure, on behalf of The Franklin Institute, that I take this opportunity to welcome all of you to this traditional occasion—the awarding of the medals for 1950 by the Institute.

"There are with us many outstanding men and women, notable for their achievements and activities in many fields. Among these we are proud to have here tonight one of the leaders of the Commonwealth of Pennsylvania. May I present to you the Honorable Gene D. Smith, Secretary of the Commonwealth of Pennsylvania."

Mr. Smith, representing Governor James H. Duff, spoke of Benjamin Franklin and the Institute and its accomplishments for the past one hundred and twenty-five years.

MR. NALLE: "Before we continue with the business of the evening, I would like to give our special thanks and appreciation to the Hostess Committee of The Franklin Institute for their assistance with the floral decorations for the evening."

* * *

MR. NALLE (*continued*): "And now—will the Sponsors for the recipients of the Medals for 1950 please take their places?"

STATED MONTHLY MEETING

MR. NALLE (*continued*): "This day marks the regular stated October meeting of The Franklin Institute. The minutes of the last Stated Meeting in May have been published in the June issue of the JOURNAL OF THE FRANKLIN INSTITUTE and, if there are no corrections or additions, they are declared approved as printed.

"It is customary to elect recipients of the Franklin Medal to Honorary Membership in The Franklin Institute. This has been done in the case of Dr. Eugene Paul Wigner, to whom the certificate of Honorary Membership will be presented this evening with the Franklin Medal."

* * *

MR. NALLE (*continued*): "Among us tonight are former recipients of Medals of The Franklin Institute. I would like to call on Mr. Lewis P. Tabor, Chairman of the Committee on Science and the Arts to give the names of these guests. Mr. Tabor."

MR. TABOR: "As these names are read, and the award each person has received mentioned chronologically in the order of the year of presentation, will each please rise and remain standing until the full roll is completed. To save time, I would like to suggest that our applause be withheld until all these are standing. Former Medalists present tonight are:

FORMER MEDALISTS ATTENDING MEDAL DAY—1950

Year	Medalist	City	Medal
1904	Dr. G. H. Clamer	Philadelphia, Pa.	Cresson
1911	Dr. Joseph S. Hepburn	Philadelphia, Pa.	Longstreth
1921	Dr. Joseph S. Hepburn	Philadelphia, Pa.	Certificate of Merit
1915	General Charles D. Young	Philadelphia, Pa.	Longstreth
1948	General Charles D. Young	Philadelphia, Pa.	Henderson
1921	Dr. L. H. Adams	Washington, D. C.	Longstreth
1923	Mr. Harry S. Parks	Pottstown, Pa.	Longstreth
1924	Mr. Thomas C. McBride	Philadelphia, Pa.	Longstreth
1926	Mr. James S. Smathers	Poughkeepsie, N. Y.	Longstreth
1932	Dr. Frank Wenner	Washington, D. C.	Wetherill
1934	Mr. Joseph G. Shryock	Philadelphia, Pa.	Certificate of Merit

1936	Mr. Peter P. G. Hall	Philadelphia, Pa.	Longstreth
1937	Dr. Rupen Eksergian	Philadelphia, Pa.	Henderson
1945	Dr. Rupen Eksergian	Philadelphia, Pa.	Levy
1937	Mr. John S. Haug	Philadelphia, Pa.	Longstreth
1937	Mr. Herbert L. Whittemore	Washington, D. C.	Longstreth
1939	Dr. N. K. Chaney	Philadelphia, Pa.	Potts
1939	Mr. E. Burke Wilford	Philadelphia, Pa.	Certificate of Merit
1940	Dr. Charles Rosenblum	Princeton, N. J.	Levy
1947	Mr. Karl P. Billner	Philadelphia, Pa.	Brown
1947	Dr. Peter J. Mulder	Shippensburg, Pa.	Certificate of Merit
1948	Mr. Ray Davis Kell	Princeton, N. J.	Ballantine
1948	Dr. Clarence A. Lovell	Summit, N. J.	Potts
1949	Dr. John W. Mauchly	Philadelphia, Pa.	Potts

MR. NALLE: "Thank you, gentlemen."

"Now, as is customary in the course of our Stated Meetings of The Franklin Institute, I would like to call on the Executive Vice President and Secretary of the Institute, Dr. Henry Butler Allen, for his report. Dr. Allen."

REPORT OF THE SECRETARY

"The Institute in 1950"

DR. ALLEN: "Mr. President, Ladies and Gentlemen: A good deal of water has gone over The Franklin Institute dam since we were together here a year ago. We feel the pulse of a sick world, and thank goodness our activities are such that we can be of prime service in these difficult days. The past year has not been devoid in progress, thanks to the keen guidance of our President, the sympathetic support of our Board, the interest of our Membership, and the complete diligence of our staff. Never have I had to do with a more unselfishly hard working group. Perhaps we all receive inspiration through the spirit of our Patron Saint, Benjamin Franklin. No more inspiring example exists of sanity and common service between man and man than that furnished by our Patron Saint, Benjamin Franklin. And we have great satisfaction in seeing proof that our Memorial to him is creating revival in awareness of the soundness of the homely virtues that he preached and lived.

"Another direct contribution by the Institute that is now increasing in tempo comes out of our industrial Laboratories for Research and Development. Besides work for industry toward more knowledge about basic materials such as asphalt and synthetic rubber, a better understanding of combustion, safe airplane landings in all kinds of weather, and aids for the blind, we are called upon for more aid for the armed services.

"And further fundamental research for peace and war is coming from our Bartol Research Foundation Laboratories, located twenty miles to the west of here in and near Swarthmore. There, during the past year, ten acres of land have been acquired, and another laboratory is now

nearly completed for the study of problems in nuclear physics with one of the world's largest particle accelerators. (I must add that this expenditure of money is possible because the Bartol Foundation has its own funds. The rest of the Institute is not able to finance such blessings.) The Bartol Laboratory too, under the auspices of the National Geographic Society has recently completed an expedition to the Hudson Bay region and has acquired fundamental information about the relation of the sun's magnetic field to the origin of cosmic rays.

"So much for research, except to invite you at your convenience to visit our laboratories. A call to my office will do the trick.

"We have not stood still in our Science Museum in the past twelve months, even though most of our new projects there have to be self-supporting. One outstanding museum activity was the first National Science Fair held in this country. This was held in June last, and the winners of awards (school children from kindergarten through high school ages) from thirteen different cities in local fairs competed here in Franklin Hall for national awards.

"Our Library, which has been assembling scientific and technical literature for one hundred and twenty-five years, now contains 134,000 volumes and 49,000 pamphlets. It is especially strong in chemistry, electricity and patents. But I should also mention its rather unique collection of rare books on early science and technology. Recently our Library endowments were helped by a generous addition to our Thorn fund.

"I should more properly wait until next year to mention that our JOURNAL OF THE FRANKLIN INSTITUTE celebrates its one hundred and twenty-fifth year of continuous publication this coming January.

"Franklin in his guise as Poor Richard once said, 'Tis hard for an empty bag to stand upright.' And knowing by experience the truth of that maxim, we in the past year have intensified our Friends of Franklin activity. This is a group (about two hundred at present) of public spirited individuals, and companies, who show their belief that The Franklin Institute is a worthy enterprise and in Franklin's adage about the empty bag—by annually contributing over and above their small membership fee.

"And during the past year we have made a fresh study of our current endowment funds, which have come to us by gift and bequest over our one hundred and twenty-six years. Exclusive of the Bartol Research Foundation fund, there are now sixty-seven such funds aggregating some $1,700,000. Only about $400,000 of these funds are completely unrestricted, although for another $300,000 the use of the income is unrestricted. The remaining funds are designated for the specific purposes of the Museum, Library, Lectures, Medals, etc., as the case may be. Of course, through the years the Institute has received with gratitude many gifts and bequests for current operating purposes and for its building

funds. Some of the deeds of gift and bequests indicate the regard in which the Institute has been held by our predecessors. We read, for example, in the simpler language of years ago, 'I have become convinced of your capacity to do good, and am desirous to promote your ability to this end.' Another bequest was made 'because of the usefulness of said Institute, and because of the association of my father and uncles with its early history.' Another read 'in appreciation of the educational value its lectures and library have been to me from the beginning of my professional career, November 1857 to 1927.' But perhaps the most interesting and the one from which the greatest lesson can be learned is the most recent and the smallest. It came the past year from a Greek immigrant who had worked as a waiter at Boothby's; who had served in the first World War in which he had been wounded; and who left his extremely small estate in small amounts to various Red Cross activities, hospitals, and a few other institutions in whose work he evidently had faith. The Institute received $191.58. This action should be an inspiration to all of us—and it is the story I should like to leave with you.

"So I add to Poor Richard's scientific reflection that an empty bag has difficulty in standing upright, a quotation pointed out to me by our Treasurer contained in the Bible under Luke 10:37: 'Go, and do thou likewise.'

"Among the pleasant events during the past year we have had two sad events: the death of Mr. Samuel Fels, Vice-President of the Institute and donor of our Planetarium bearing his name; and more recently, that of Mr. E. Paul du Pont, a newer member of our Board.

"Mr. President, Medalists for 1950, Past Medalists, Members and Friends: Perhaps I have taken too long in telling you about some of the highlights of the past twelve months. We shall have more to tell you if you will come back next October. But meanwhile let us see you at the Institute whenever you can get here. And if your visit is in the summer or fall, be sure to see our garden created through our faithful Hostess Committee alongside our red brick Laboratory building in the rear. We hope our Medalists will always look upon the Institute as their home when in this city; and that their visits will be frequent."

MR. NALLE: "Thank you, Dr. Allen."

* * *

MR. NALLE (*continued*): "For one hundred and twenty-six years now, reports such as Dr. Allen's have been made on the affairs of The Franklin Institute.

"During those one hundred and twenty-six years extraordinary changes have been made in this world of ours. The scientific, religious, economic, political and social phases of the entire human race have all undergone forceful, and in most cases, beneficial alterations. It can very truly be said that science and the farsighted and penetrative minds

of students of science have been mainly responsible for most of these transformations.

"Today the words *new*, *modern* and *up-to-date*, continue to be important factors in our thinking and actions. They are in constant use in our everyday conversation. To a great extent this is constructive and yet, were we to pause and reflect, we would realize and acknowledge that many of the more important benefits and advantages we enjoy today are the results of the toil, trial and endeavor of those who have gone before us.

"Indeed, there are certain traditions and occasions which require no change, nor do they need to be modernized. Such a one is the ceremony for which we are gathered tonight. The Committee on Science and the Arts of The Franklin Institute is charged with the duty of searching out and rewarding meritorious invention and discovery in many fields of science. They discharge their duties in a most serious, able and conscientious manner.

"This is not only the oldest Committee of the Institute but also the best known. It has maintained for more than a century the high standard of the awards made by the Institute. I want to convey to them tonight the thanks and appreciation of the Board of Managers.

"We are assembled for another Medal Day, to carry out the findings of the Committee on Science and the Arts by recognizing and paying tribute to the valuable achievements of certain outstanding men of science.

"Each Medalist, when he comes to the Institute to receive his award, is sponsored by a member of the Sub-Committee which recommended him to the Science and Arts Committee. Generally this Sponsor is the Chairman of the Sub-Committee. The award of each Medal is accompanied by a copy of the report on the case together with a certificate indicating the citation.

"Tonight it is a matter of great regret that three of our Medalists cannot be with us in person. Dr. Eugene Paul Wigner, the Thomas D. Jones Professor of Mathematical Physics, Princeton University, who is at this time in Europe, is being represented by his wife, Mary Wheeler Wigner, Ph.D., a physicist in her own right and a former Loomis Fellow at Yale. She will receive the Franklin Medal for Dr. Wigner.

"Dr. Basil F. J. Schonland of South Africa, and Monsieur Eugène Freyssinet of Paris, France, have also had to designate proxies to receive their Medals. These gentlemen are Mr. D. R. Masson, Scientific Attaché to the Embassy of the Union of South Africa, and Mr. R. M. Dubois of the Inter-Continental Equipment Corporation. The two Medals which will be presented to them here tonight will then be sent abroad to the recipients.

"Now we shall call upon the various Sponsors to present their candidates."

PRESENTATION OF THE POTTS MEDAL
(Founded in 1906)

(To Merle A. Tuve, Director, Department of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, D. C.)

MR. NALLE: "The Chair recognizes Dr. James Barnes."

DR. BARNES: "Mr. President, I present Merle A. Tuve for an award.

"Dr. Tuve is Director of the Department of Terrestrial Magnetism, Carnegie Institution, Washington, D. C. He has had a distinguished career in physics having, amongst other things, contributed notably to the development of high energy nuclear particle accelerators. At the beginning of World War II, he devoted himself to the solution of a problem of pressing importance, the development of an artillery fuze

Merle Antony Tuve

which would fire when the projectile passed by a target in a near miss. This type of fuze, now known as the Proximity Fuze, had been sought over many years by workers of many nationalities without success. Dr. Tuve presented plans for such a device and then carried these plans into execution in ample time to permit Proximity Fuzes to exert an important influence in the land fighting in Europe and in the sea fighting in the Orient. In the course of this program a multitude of scientific and engineering problems had to be solved and a host of administrative and manufacturing difficulties had to be overcome. The Proximity Fuze is now recognized to be one of the two or three most remarkable scientific achievements of World War II.

"I present Merle A. Tuve, of Washington, D. C., as a candidate for a

Howard N. Potts Medal, 'In recognition of the scientific insight exhibited in the conception of the VT Proximity Fuze and of the administrative ability and practical good judgment shown in the supervision of its development and engineering design.' "

MR. NALLE: "Dr. Tuve, by virtue of the power vested in me as President of The Franklin Institute of the State of Pennsylvania, I present to you this Howard N. Potts Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE HENDERSON MEDAL
(Founded in 1924)

(To Paul Walter Kiefer, Chief Engineer, Equipment; New York Central System, New York, New York.)

Paul Walter Kiefer

MR. NALLE: "The Chair recognizes Dr. Rupen Eksergian."

DR. EKSERGIAN: "Mr. President, I present Paul Walter Kiefer for an award.

"Mr. Kiefer's contributions to railway engineering have been along three directions.

"His work in modernizing the steam locomotive by coordinating important design features through research and test, resulted in more than doubling its mileage availability together with increased over-all performance. The famous 'Niagara' developed 6600 hp. and reaches an outstanding availability of 288,000 miles per annum, closely approximating that of the diesel-electric, along with reduced annual operating cost per mile.

"His contributions to car design resulted in basic improvements to the modern box car, through increased impact strength and better protection to loading. He also contributed to improvements of trucks, draft gear design and passenger car riding comfort.

"Mr. Kiefer's work on system analysis of motive power is outstanding. In a book on the *Practical Evaluation of Railroad Motive Power*, he lists four basic yardsticks for appraising modern locomotives. He then compares the relative merits of the reciprocating steam, the steam and gas turbine, the straight electric and, finally, the diesel-electric locomotive. The methods of analysis offer a procedure for appraising new developments in locomotive design.

"I present Paul Walter Kiefer, of New York, New York, as a candidate for the George R. Henderson Medal, 'For his contribution to the development of the modern steam locomotive, resulting in great advance in its design and performance; for his contribution to the development of modern freight and passenger car equipment producing better riding qualities and leading to smoother operation; and for his development and use of methods of analysis and appraisal of the design and utilization of locomotive and car equipment which made possible great improvement in the over-all, or system operation of this equipment.' "

MR. NALLE: "Mr. Kiefer, by the same authority, I present to you this George R. Henderson Medal and the Certificate and Report which accompany it."

PRESENTATION OF THE WETHERILL MEDALS
(Founded in 1925)

(To Donald W. Kerst, University of Illinois, Urbana, Illinois; Kenneth C. D. Hickman, Rochester, New York; Russell H. Varian, President, Varian Associates, San Carlos, California; and Sigurd F. Varian, Director of Development Laboratory, Varian Associates, San Carlos, California.)

MR. NALLE: "The Chair now recognizes Mr. John H. Neher."

MR. NEHER: "Mr. President, I present Donald William Kerst for an award.

"Since the first transmutation of the elements was accomplished in 1919, scientists have constantly striven to develop new and better devices for the bombardment of the atom, for it is by this means that transmutation and allied phenomena are caused to occur.

"When the cyclotron was successfully employed in bombardment by positive ions, scientists suggested comparable means for accelerating electrons in a circular orbit by employing the principle of magnetic induction. Several investigators subsequently constructed devices utilizing these principles but the intensity of the electron beam obtained was not satisfactory.

"Dr. Kerst then became interested in the device which he subsequently developed to a state where a sufficient beam intensity was obtained, and which he called the 'Betatron,' since this is a device for generating high-velocity electrons, or beta rays. It was Dr. Kerst's thorough analysis of the principles involved, as well as his development of the equations for transient phenomena in the magnetic field, and his practical application of these principles which made his the first successful working model of this device.

"The Betatron has proved itself extremely valuable not only in nuclear research but also in the field of medicine, specifically in the treatment of cancer.

Donald W. Kerst

"I present Donald William Kerst, of Urbana, Illinois, as a candidate for a John Price Wetherill Medal, 'In consideration of his contribution to the theory of the Betatron and the practical application of the theory to the construction of the first satisfactory workable machine, and his subsequent work in this field.'"

MR. NALLE: "Dr. Kerst, by the same authority, I present to you this John Price Wetherill Medal and the Certificate and Report which accompany it."

"The Chair recognizes Mr. W. H. Fulweiler."

MR. FULWEILER: "Mr. President, I present Kenneth C. D. Hickman for an award.

"A popular aim today is to get something for nothing. Our Medalist has virtually accomplished this by recovering valuable products from hitherto unusable raw materials.

"The commercial application of the process he has developed has yielded abundant supplies of Vitamins A and E from fish oils and waste products.

"Essentially molecular distillation is carried out under conditions such that the molecules of the substance pass directly from the heated surface to the condenser without collision.

"This requires operation at a very low pressure of the order of less than one ten thousandths of an inch of mercury. The construction of high capacity vacuum pumps was an essential feature of the process.

Kenneth C. D. Hickman

"Many organic materials are very sensitive to heat. The Vitamins are particularly susceptible.

"By reducing the time of exposure from a matter of hours to seconds or even fractions of a second, the destructive effect may be reduced by a factor of a million or more.

"This has been accomplished in the centrifugal still where the material is heated in a very thin film formed on the surface of a rapidly rotating cone. These cones may vary from five to ten inches in laboratory apparatus to as many feet in commercial practice.

"This development required many men, many years, and many dollars.

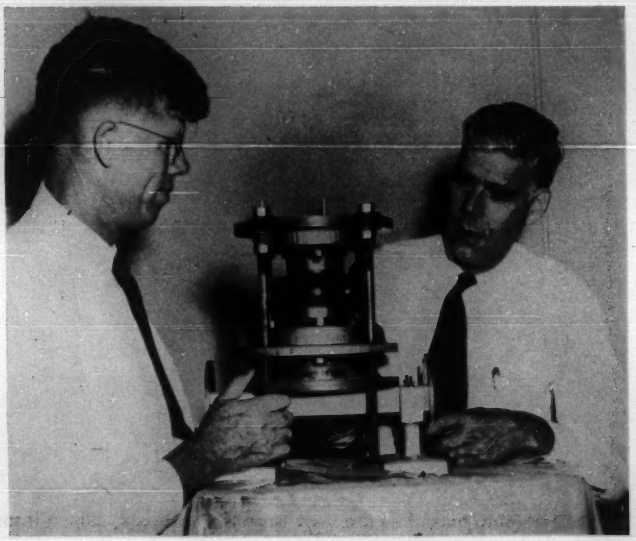
"I present Kenneth C. D. Hickman, of Rochester, New York, as a candidate for a John Price Wetherill Medal, 'In consideration of his ingenuity and leadership resulting in development of improved means in producing high vacua by the use of oil diffusion pumps; for significant

advances resulting from the design of the centrifugal still and for his commercial adaptation of these improved practices to the recovery of Vitamins A and E from hitherto unusable raw materials.' "

MR. NALLE: "Dr. Hickman, by the same authority, I present to you this John Price Wetherill Medal and the Certificate and Report which accompany it."

"The Chair now recognizes Mr. Walter C. Wagner."

MR. WAGNER: "Mr. President, I present Sigurd F. Varian and Russell H. Varian for awards.



Russell H. Varian and Sigurd F. Varian

"During the later thirties, a few foresighted people recognized the tremendous military importance of being able to detect and locate enemy aircraft at a distance. Much of this work was conducted in government laboratories, or by government sponsorship in a few other laboratories, under extremely strict security regulations.

"Among these foresighted people were the Varian brothers, who recognized that one of the keys to success in the aircraft location problem was the development of a device for producing what are now known as microwaves.

"Working independently, with only advice from friends in a college laboratory, they were successful in developing the klystron, an extremely valuable generator of microwaves, which was used in almost all of the

microwave radar equipment which played so great a part in our military activities during World War II, and which is now finding extensive peacetime applications in the field of communications.

"I present Sigurd F. Varian and Russell H. Varian, both of San Carlos, California, as candidates for a John Price Wetherill Medal each, 'In recognition of their foresight in anticipating the need for a converter of electromagnetic energy in a hitherto unexplored high frequency region of the spectrum, their energy and technical insight in developing for this purpose, a practical device, the klystron, which has opened up new fields in communications applications and research.'"

MR. NALLE: "To you, Mr. Varian and to you, Dr. Varian, by the same authority, I present these John Price Wetherill Medals and the Certificates and Reports which accompany them."

Gustave Magnel

PRESENTATION OF THE BROWN MEDALS
(Founded in 1938)

(To Eugène Freyssinet, Societé Technique pour l'Utilization de la Précontrainte, Paris, France; and Gustave Magnel, University of Ghent, Ghent, Belgium.)

MR. NALLE: "The Chair recognizes Mr. Coleman Sellers, 3rd."

MR. SELLERS: "Mr. President, I present Eugène Freyssinet, *in absentia*, and Gustave Magnel for awards.

"The tensile strength of concrete is very low, a fact which has prevented it from being used in any member in which tensile stresses are

expected to occur unless it is reinforced with steel bars. The design is then based on the tensile strength of the steel.

"On the other hand, the compressive strength of concrete is high but until the advent of prestressed concrete, we have not been able to take full advantage of this inherent characteristic.

"Prestressed concrete, as it is now made, carries compressive stresses so great that the beam or structure will never be in tension. These stresses are induced by means of high strength steel wires.

"While prestressed concrete is not new in theory, it remained for our Medalists to make it a practical reality. They have taught the world how to design and fabricate structures of this new and valuable building material.

"The result of their work is shown in many bridges, industrial buildings, airport hangars, dams, tanks, etc. in many European countries as well as Africa and South America. The Walnut Lane bridge in Fairmount Park, now under construction, is the first example in this country of a structure using prestressed concrete beams.

"I present Eugène Freyssinet, of Paris, France, *in absentia*, and Gustave Magnel, of Ghent, Belgium, to each of whom has been awarded a Frank P. Brown Medal, 'For their outstanding development of engineering and technique for prestressed concrete members resulting in radical improvements in the design and construction of bridges, buildings and other structures.' "

MR. NALLE: "By the same authority, Mr. Dubois, I present to Monsieur Freyssinet through you, and to you, Professor Magnel, these Frank P. Brown Medals and the Certificates and Reports which accompany them."

PRESENTATION OF THE CLAMER MEDAL
(Founded in 1943)

(To Charles Sanborn Barrett, Institute for the Study of Metals, University of Chicago, Chicago, Illinois.)

MR. NALLE: "The Chair now recognizes Mr. Francis B. Foley."

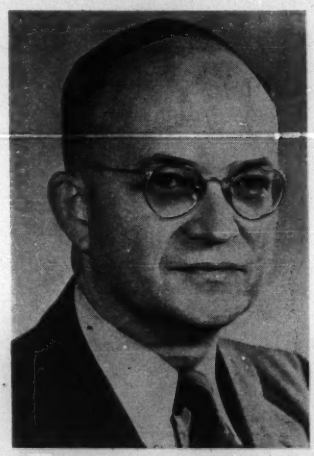
MR. FOLEY: "Mr. President, I present Charles Sanborn Barrett for an award.

"The study of the nature of metals and alloys rests largely on an accurate knowledge of their crystalline structure and it is through that knowledge that new alloys are developed and the behavior of the metals and alloys of commerce under service conditions is understood and controlled. The short wave length characteristics of X-ray have made them peculiarly suited to the determination of the position and spacing of atoms in crystals. Foremost in the application of short-wave emissions to the study of metallic crystals and the effect on them of deformation, strain-aging and recovery upon annealing after cold-working has been Dr. Barrett who, over a period of more than twenty years, during which

time he has discovered a method of developing microstructure by means of X-rays, has made scholarly contributions of great value to our knowledge of the behavior of metallic crystals.

"I present Charles Sanborn Barrett of Chicago, Illinois, as a candidate for the Francis J. Clamer Medal, 'In recognition of his outstanding contribution to the knowledge of crystal structures and of the effects of deformation and aging on various metallic crystals.' "

MR. NALLE: "Dr. Barrett, by the same authority, I present to you this Francis J. Clamer Medal and the Certificate and Report which accompany it."



Charles Sanborn Barrett

PRESENTATION OF THE CRESSON MEDAL
(Founded in 1848)

(To Basil F. J. Schonland, South African Council for Scientific and Industrial Research, Johannesburg, South Africa.)

MR. NALLE: "The Chair recognizes Dr. Richard M. Sutton. Mr. D. R. Masson, Scientific Attaché to the Embassy of the Union of South Africa, has come here this evening as proxy for Dr. Schonland.

DR. SUTTON: "Mr. President, I present, *in absentia*, Basil F. J. Schonland for an award.

"For more than a quarter century, Professor Schonland has made the study of Atmospheric Electricity his major interest, and his work has won world-wide acclaim. Particularly as an investigator of Lightning, he has furnished us with clear evidence of many of its mysteries. The results of his studies, accomplished by modern photographic and

electronic means, would warm the heart of our own Benjamin Franklin who also, and for good reason, made a name in Lightning! Professor Schonland has succeeded in dissecting this grand but frightening split-second event from the skies and has given it clear interpretation in terms of modern atomic physics.

"It seems fitting to note that this distinguished Medalist was, during the recent war, a Brigadier in Field-Marshall Montgomery's main Headquarters staff as chief of Operational Research for the British Army.

"I present, *in absentia*, Basil F. J. Schonland, of Johannesburg, South Africa, as a candidate for an Elliott Cresson Medal, 'In consideration of

Basil F. J. Schonland

his extended work in the field of Atmospheric Electricity and of his expermental investigations into Lightning, and of his conclusions regarding the mechanism of the Lightning Discharge as it occurs in nature.' "

MR. NALLE: "Mr. Masson, by the same authority, I take pleasure in presenting to Dr. Schonland, through you, this Elliott Cresson Medal, and the Certificate and Report which accompany it."

PRESENTATION OF THE FRANKLIN MEDAL
(Founded in 1914)

(To Eugene P. Wigner, Thomas D. Jones Professor of Mathematical Physics, Princeton University, Princeton, New Jersey.)

MR. NALLE: "The Chair now recognizes the final Sponsor of the evening, Dr. Nevin E. Funk, to present the candidate for the Franklin Medal."

DR. FUNK: "Mr. President, I have the honor to present, *in absentia*, Eugene Paul Wigner for an award.

"It may be said that one of the tasks of theoretical physics is to express in an understandable formula some relationship or structure existing in physical reality. In the field of classical physics, it has sometimes been useful to imagine a physical model of the physical reality which is being studied. Having imagined such a physical model, the classical physicist then constructs a mathematical analogue describing, by formula and text, how the physical model works. For the many cases in which the concepts of classical physics are valid, the model has been sufficiently close to reality so that such a procedure has been very satisfactory. In modern atomic physics, the things being studied are very small, move very rapidly, and are subjected to very great forces.

Eugene Paul Wigner

They are so different from things that we can see or hear or touch that it is sometimes difficult, if not impossible, to imagine a physical model which could behave as they seem to do. When dealing with such concepts, mathematical physicists pass over the step of imagining a physical model and make mathematical constructs which represent, as nearly as may be, the relationships or structure of the physical reality being studied. Dr. Wigner has added much to the mathematics which can be used to describe the phenomena of atomic physics. He has also applied these new mathematical concepts to many problems in that field. Furthermore, his skill was used in making the necessary calculations for designing the atomic pile at Hanford during World War II.

"Mr. President, I present, *in absentia*, Eugene Paul Wigner, of Princeton University, as a candidate for the Franklin Medal, 'In consideration of his work in the application of Group Theory to Quantum Mechanics and his work on many fundamental problems of Theoretical Physics.' "

MR. NALLE: "By the authority vested in me as President of The Franklin Institute, I present, *in absentia*, to Dr. Eugene Paul Wigner, through his wife, Dr. Mary Wheeler Wigner, this Franklin Medal with its accompanying Certificate, Report and a Certificate of Honorary Membership in The Franklin Institute.

"Dr. Allen, will you, as Secretary of The Franklin Institute, please assist Mr. Dubois and Mr. Masson in any arrangements necessary to send these awards to their recipients in France and in South Africa?"

DR. ALLEN: "Yes indeed, Mr. Nalle."

Upon the conclusion of each award, the recipient briefly expressed his appreciation.

MR. NALLE: "The Sponsors may now resume their places.

"It is now my pleasure to present to you a physicist in her own right, who will read for us the paper entitled 'The Role of Mathematical Methods in Physical Theories' which her husband, Dr. Eugene Paul Wigner, the 1950 Franklin Medalist, has prepared for this evening. We have investigated her record in 'American Men of Science' and find the following: She is originally from Fair Haven, Vermont. She received a Bachelor of Arts degree from Wellesley College in 1923, she was a Loomis Fellow at Yale from 1929 to 1930, from which university she received a degree of Doctor of Philosophy in 1932. She was instructor and assistant professor of physics at Vassar College from 1930 to 1942. Perhaps I can best sum up the whole situation by the slogan, 'Never underestimate the power of a woman.' With great pleasure, I introduce Dr. Mary Wheeler Wigner.*"

Mr. Nalle, after Dr. Wigner had read this paper, conveyed to her the thanks of the Institute, and dismised the assembly with a cordial word and declared the meeting adjourned.

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* See page 477 for the text of this paper.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

November 15, 1950

The November stated meeting of The Franklin Institute was held in the Lecture Hall on Wednesday evening, November 15, 1950, with approximately 125 persons attending.

The President called the meeting to order at 8:30 P.M., and reported that the minutes of the stated meeting for October will be presented for approval at the December meeting.

The Secretary reported that the following elections to membership have been recorded since the stated meeting of April 19, 1950:

Active	263
Associate	198
Student	92

The total Institute membership as of October 31, was 5,938.

The Secretary announced that on November 27 at 2:30 P.M. there will be dedicated the New Laboratory for Nuclear Physics which houses the Van de Graaff Accelerator. The Laboratory is on Baltimore Pike at Church Road, Springfield Township.

Also announced by the Secretary was the information that the 1950 series of The James Mapes Dodge Christmas Week Lectures for young people entitled, "Touring the Universe," will be given on December 28 and 29 at 4:00 P.M. at the Institute. The speaker will be Dr. H. R. Kingston, Dean of Arts and Sciences of the University of Western Ontario. Dr. Kingston is past president of the Royal Astronomical Society of Canada and is famous for his popular lectures and demonstrations in astronomy.

The President then introduced the guest speaker, Rear Admiral Milo F. Draemel, U. S. N. (Retired) whose subject was the "Protection and Conservation of Pennsylvania's Water Resources." Admiral Draemel, who is now Secretary of the Department of Forests and Waters, Commonwealth of Pennsylvania, discussed the importance of protecting and conserving our water resources so that America may remain the land of opportunity for the generations to come. He told of the active pollution abatement program and showed slides which portrayed the condition of the Schuylkill River before the program was begun and scenes of the work which has been accomplished. A colored film, "Dangerous Waters," which presented clearly the reclamation work, was shown.

Admiral Draemel was given a rising vote of thanks as the meeting adjourned.

HENRY B. ALLEN
Secretary

JOURNAL OF THE FRANKLIN INSTITUTE

The January 1951 issue of the JOURNAL will commemorate the 125th anniversary of its publication. Major Thomas Coulson, Director of Museum Research for the Institute, has prepared a brief history of the JOURNAL, outlining *past* achievements. A special symposium, entitled "Science and Tomorrow," offers papers by fifteen outstanding authorities who will examine their own fields for possible and probable *future* developments. The editors are honored to announce the following papers for our 125th anniversary issue:

SCIENCE AND TOMORROW

Basic Physical Sciences

Astronomy	Dr. Otto Struve
Chemistry	Dr. Harold C. Urey
Mathematics	Dr. Marston Morse
Physics	Dr. I. I. Rabi

Applied Sciences and Technology

Aeronautics	Dr. Jerome Hunsaker
Communications	Dr. Oliver E. Buckley
Controlled Mechanisms	Dr. Norbert Wiener
Electronics	Dr. V. K. Zworykin
Materials	Dr. Wallace R. Brode
Medicine	Dr. Detlev Bronk
Nutrition	Dr. Henry C. Sherman
Standardization	Dr. E. U. Condon
Transportation	Dr. Charles Kettering

Special Topics

Cosmic Rays	Dr. W. F. G. Swann
Solid State Physics	Dr. Frederick Seitz

Concluding Paper

Dr. W. F. G. Swann

LIBRARY

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Fridays and Saturdays from 9 A.M. until 5 P.M.; Wednesdays and Thursdays from 2 P.M. until 10 P.M.

RECENT ADDITIONS

AGRICULTURE

ISRAELSEN, ORSON WINSO. Irrigation Principles and Practices. Ed. 2. 1950.

BIBLIOGRAPHY

AMERICAN SOCIETY OF CIVIL ENGINEERS. Selected Bibliography on Soil Mechanics. 1940.

AMERICAN SOCIETY OF CIVIL ENGINEERS. Selected Bibliography on Construction Methods. 1930.

ATWOOD, ALICE CARY. Peat; a Contribution Towards a Bibliography of the American Literature Through 1925. 1926.

BESTERMAN, THEODORE. World Bibliography of Bibliographies. Vol. 1-3. 1950.
CHARLES, MICHEL. Catalogue de la Bibliothèque Scientifique, Historique at Litteraire de Feu. 1881.
INSTITUTO NACIONAL DEL LIBRO ESPAÑOL. Ensayo de Bibliografía Marítima Español. 1943.
JÜRGENS, ADOLF. Ergebnisse Deutscher Wissenschaft. 1939.

CHEMISTRY AND CHEMICAL TECHNOLOGY

BANGHAM, D. H. Progress in Coal Science. Vol. 1. 1950.
FAITH, WILLIAM LAWRENCE, KEYES, DONALD BABCOCK AND CLARK, RONALD L. Industrial Chemicals. 1950.
GATTERMANN, LUDWIG. Die Praxis des Organischen Chemikers. Ed. 4. 1900.
GMELIN, LEOPOLD. Gmelins Handbuch der Anorganischen Chemie. 1929–1943.
HAMERSLAG, FRANK E. The Technology and Chemistry of Alkaloids. 1950.
HOFMANN, KARL A. Lehrbuch der Anorganischen Chemie. Ed. 2. 1919.
HÜCKEL, WALTER. Structural Chemistry of Inorganic Compounds. Vol. 1. 1950.
LAIDER, KEITH. Chemical Kinetics. 1950.
MOISSAN, HENRI. Das Fluor. 1900.
MOORE, WALTER J. Physical Chemistry. 1950.
NACHTRIEB, NORMAN HARRY. Principles and Practice of Spectroanalysis. 1950.
PFEIFFER, J. P. The Properties of Asphaltic Bitumen. 1950.
OST, HERMANN. Lehrbuch der Chemischen Technologie. Ed. 10. 1919.
PLOTNIKOW, IVAN. Grundniss der Photochemie. 1923.
RICHTER, HOLMES. Laboratory Manual of Elementary Organic Chemistry. 1940.
RINNE, FRIEDRICH. Das Fein Bauliche Wesen der Materie nach dem Vorbilde der Kristalle. 1922.
ROSSINI, FREDERICK DOMINIC. Chemical Thermodynamics. 1950.
WILKINSON, D. H. Ionization Chambers and Counters. 1950.

DICTIONARIES

THOMPSON, R. N. AND HAIM, G. Welding Dictionary, French-German-Spanish-English. 1950.

ELECTRICITY AND ELECTRICAL ENGINEERING

DAMMERS, B. G. AND OTHERS. Application of the Electronic Valve in Radio Receivers and Amplifiers. 1950.
JOHNSON, WALTER CURTIS. Transmission Lines and Networks. 1950.
KRAUS, JOHN DANIEL. Antennas. Ed. 2. 1950.
SOUTHWORTH, GEORGE C. Principles and Applications of Waveguide Transmission. 1950.

ELECTRONICS

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS AND INSTITUTE OF RADIO ENGINEERS. Second Joint AIEE-IRE Conference on Electronics in Nucleonics and Medicine. 1950.
DOUGLAS, ALAN. The Electronic Musical Instrument Manual. 1949.

ENGINEERING

DUNHAM, CLARENCE W. Foundations of Structures. 1950.

GEOLOGY

RANKAMA, KALERVO AND SAHAMA, THURE GEORG. Geochemistry. 1950.

INDUSTRIAL MANAGEMENT

APPLE, JAMES MACGREGOR. Plant Layout and Materials Handling. 1950.

MANUFACTURE

BOKMA, F. T. AND KRAAY, G. M. Rubber en Latex. 1949.
BROCK, ALAN S. A History of Fireworks. 1949.

CAMPBELL, JAMES S., JR. Casting and Forming Processes. 1950.
DAVIDSOHN, J. AND DAVIDSOHN, A. Polishes. 1949.
EBERT, HERMANN. Anleitung zum Glasblasen. Ed. 4. 1912.

MATHEMATICS

CARNAP, RUDOLF. Logical Foundations of Probability. 1950.
COWLES COMMISSION FOR RESEARCH IN ECONOMICS. Statistical Inference in Dynamic Economic Models. 1950.
FISHER, RONALD, AYLMER. Contributions to Mathematical Statistics. 1950.
HEIBERG, J. L. AND ZEUTHEN, H. G. Eine Neue Schrift des Archimedes. 1907.
HOLMES, CECIL THOMAS. Calculus and Analytical Geometry. 1950.
MAZIARZ, EDWARD A. The Philosophy of Mathematics. 1950.
WALD, ABRAHAM. Statistical Decision Functions. 1950.
ZYGMUND, ANTONI. Contributions to Fourier Analysis. 1950.

MECHANICAL ENGINEERING

BOKS, D. J. Pompen. 1948.
BOWDEN, F. P. AND TABOR, D. The Friction and Lubrication of Solids. 1950.
CALLACOTT, RALPH. Mechanical Vibrations. 1947.
DOYLE, LAWRENCE E. Tool Engineering. 1950.
HOUGHTON, P. S. The Milling Machine. 1948.
LORAIN, PIERRE. Notions sur les Machines à Vapeur. 1947.
OBERT, EDWARD FREDERIC. Internal Combustion Engines. Ed. 2. 1950.
POHL, ROBERT WICHARD. Einführung in die Mechanik und Akustik. Ed. 2. 1931.
SMITH, GEORGE GEOFFREY. The Modern Diesel. Ed. 4. 1949.
WALSHAW, T. D. Diesel Engine Design. 1949.
WOLF, KARL. Lehrbuch der Technischen Mechanik Starrer System. Ed. 3. 1947.

METEOROLOGY

SCHÜEPP, WALTER. Die Bestimmung der Komponenten der Atmosphärischen Trübung aus Aktinometer Messungen. 1949.

PHYSICS

BERG, RALPH ROGER. Heat Transfer to Fluids in Coiled Tubing in the Streamline Flow Region. 1950.
BIRKHOFF, GARRETT. Hydrodynamics. 1950.
BUFF, HEINRICH. Lehrbuch der Physikalischen Mechanik. Vol. 1–2. 1873–1874.
CULLUM, D. J. W. The Practical Application of Acoustic Principles. 1949.
DAVIDSON, MARTIN. Introduction to Dynamics. 1949.
GANT, P. Mechanics. Vol. 1–3. 1949.
KERN, DONALD QUENTIN. Process Heat Transfer. 1950.
LEWITT, ERNEST HENRY. Thermodynamics Applied to Heat Engines. Ed. 4. 1949.
NADAI, ARPAD LUDWIG. Theory of Flow and Fracture of Solids. Ed. 2. 1950.
ROCARD, Y. Dynamique générale des Vibrations. Ed. 2. 1949.
SEARS, FRANCIS WESTON. An Introduction to Thermodynamics, the Kinetic Theory of Gases and Statistical Mechanics. 1950.
STEPHENS, R. W. B. AND BATE, A. E. Wave Motion and Sound. 1950.
TEICHMANN, H. Komprimierte und Verflüssigte Gase. 1908.
VOLKMANN, WILHELM. Praxis der Linsenoptik in Eifachen Versuchen zur Erlauterung und Prüfung Optischer Instrumente. 1910.
WILKES, GORDON BALL. Heat Insulation. 1950.

PLASTICS

ENGEL, H. C., HEMMING, CHARLES B. AND MERRIMAN, H. R. Structural Plastics. 1950.

SANITARY ENGINEERING

BABBITT, HAROLD EATON. Plumbing. Ed. 2. 1950.

SCIENCE

BURCHARD, JOHN ELY. Mid-century. 1950.
FRANK, PHILIPP. Modern Science and its Philosophy. 1949.
HERTZ, DAVID BENDEL. The Theory and Practice of Industrial Research. 1950.
WIGHTMAN, WILLIAM P. D. The Growth of Scientific Ideas. 1950.

SUGAR

PRINGSHEIM, HANS. Die Polysacchande. Ed. 2. 1923.

TEXTILES

CHAMBERLAIN, JOHN. Knitting Mathematics and Mechanisms. Ed. 3. 1950.

MUSEUM

The two most ancient sciences were alchemy and astrology. Our modern science of chemistry was founded upon the work of the alchemists. Largely because the alchemist was an experimentalist his science never fossilized, it was dynamic and recognized change. Modern astronomy, on the other hand, has had to develop independently of astrology. The latter "science" was founded upon the conviction that the universe was constructed of a series of concentric spheres with the earth at their center. The outer sphere was supposed to exercise an occult influence upon the inner sphere. When the modern conception of a heliocentric system was introduced, together with the idea of an infinite space where there could be neither outer nor inner spheres, the astrologer was abruptly deprived of all basis for his beliefs. Some deluded people continue to believe in a fallacy that was exploded nearly four centuries ago.

A great many fallacies were exposed at about the time when the Copernican system was being established through the aid of the telescope but some of the ideas of the ancients were not fallacies, they were truths. For example, the fundamental idea of time as based upon the motions of the earth is of very early date, but the ambitions of various Roman emperors to perpetuate their names threw the calendar into confusion. Not only the months but the numbering of the years became confused, so that there is considerable doubt that the basic year of Christian reckoning is correct.

The dispute about the actual year of the Nativity affords opportunity for showing in the Planetarium each year a rare conjunction of planets which occurred in the year 7 B.C. Reasons are given for supposing that this may have been the year of the Nativity. The scientific approach to the subject is not permitted to obscure the devotional attitude. While certain celestial phenomena are admitted to have occurred which may have an unusual significance, the appearance of the Star of Bethlehem remains the focus of interest at the time of the Nativity. Each year at this time the Planetarium demonstration is devoted to the study of the Christmas skies, taking us each back through the ages to the time when shepherds, wise men, and kings looked into the heavens and found there a symbol of faith. Through the medium of this unique instrument a program of absorbing interest has been prepared in which appropriate music and ingenious applications of light assist the onlooker to recapture the spirit of the season.

The story of the Star of Bethlehem as told in the Fels Planetarium never fails to renew its interest upon the thousands of people who have seen it. Visitors come year after year and express the opinion that its freshness does not fade. To those who see the demonstration for the first time there comes the realization that here is a new method of replenishing faith in old traditions and investing them with renewed vigor. This annual demonstration has become so much of an established feature of the season in Philadelphia that it can be said truly of those who attend regularly: "It was always said of him, that he knew how to keep Christmas well," as Charles Dickens said.

MEMBERSHIP

ACTIVE MEMBERS ELECTED AT THE MEETING OF THE BOARD OF MANAGERS, NOVEMBER 15, 1950

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William H. Bokum

NECROLOGY

Robert J. Bennett '42
Paul G. Brown '36
Mrs. Robert Dechert '35
Merritt L. Fernald, D.C.L., D.Sc. '38
Alphonsus L. Hauck, Jr. '49
C. H. Knauer, M.D. '49
R. Judson Riggs '42

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

***Abstract of* Protection of Mice against X-Radiation by Thiourea.**—G. LIMPEROS AND W. A. MOSHER (*Science*, **112**, 86 (1950)). Thiourea, injected intraperitoneally (2140 mg./kg.) immediately before X-radiation or given in the drinking water (1%) for a period of six days before X-radiation with 600 r, significantly reduced the mortality of X-rayed mice. Of the control mice, 68% survived, whereas 86% of the mice given thiourea in their drinking water survived. The survival rate of the mice injected with thiourea before irradiation was 35.2%, for the mice injected after irradiation, 7%, and for the controls, 2%. It is believed that the protective effect of thiourea is due to the protection afforded to vital cellular constituents such as nucleic acid, cysteine, glutathione, and sulfhydryl-containing enzymes.

***Abstract of* The *In Vivo* Synthesis of Labile Methyl Groups.**—S. JONSSON AND W. A. MOSHER (*J. Am. Chem. Soc.*, **72**, 3316 (1950)). Serine labeled with C^{14} in the β-position was given intraperitoneally to male rats over a period of five days. The finding of radioactivity in the methyl groups of choline isolated from the liver demonstrated the *in vivo* synthesis of labile methyl groups from the β-carbon of serine and hence from the α-carbon of glycine. There was indication that serine, at least in part, was converted to ethanolamine. The incorporation of the methyl group of radioactive formaldehyde into choline was also observed.

***Abstract of* Factors Necessary for Maximum Growth of *Clostridium bifermentans*.**—L. DS. SMITH AND H. C. DOUGLAS (*J. Bact.*, **60**, 9 (1950)). The maximum growth of *Clostridium bifermentans* in a medium composed of "vitamin-free" acid-hydrolyzed casein, glucose, and inorganic salts was found to be dependent upon added biotin, nicotinic acid or amide, pantothenic acid, and pyridoxal or pyridoxamine. Some strains did not require biotin and some strains did not require pyridoxamine.

The maximum growth of *C. bifermentans* in an amino acid medium was found to be dependent upon some unknown substance, probably a peptide.

BOOK REVIEWS

THE BIOCHEMISTRY OF B VITAMINS, by Roger J. Williams, Robert E. Eakin, Ernest Beerstecher, Jr., and William Shive. 741 pages, diagrams and tables, 15 × 23 cm. American Chemical Society Monograph No. 110. New York, Reinhold Publishing Co., 1950. Price, $10.00.

This, the latest book of many on the B vitamins, is made up of four separate but closely related and well integrated parts, each by one of the four authors. Section A (R. J. Williams) covers the basic information on the nature, occurrence, assay methods, and biological origin of the B vitamins. Section B (R. E. Eakin) deals with the catalytic, or enzyme-like, functions of the B vitamins. Section C (E. Beerstecher) is primarily concerned with plant and animal requirements of B vitamins. Section D (W. Shive) attempts to develop the actual biochemical pathways involved in B vitamin metabolism and antagonisms to these pathways.

In evaluating this monograph the significant question is: "Where does this work stand in comparison with the many other fine books already covering the B vitamin field?" The answer is easy: This book stands at the head of the list. Not just because it is the most recent, or well written (although some parts are superb in this respect), or well documentated with the critical references, or because of the experience and standing of the authors. In a rapidly moving field, this book stands with one foot in the past and the other in the future. It offers adequate and critical treatment of past work in the B vitamin field. The newcomer to the field, the student, will find its treatment of the historical facts fair and sufficient but free of the usual mass of now meaningless detail. The experienced worker in Biochemistry will find the B vitamins properly oriented and considered, not as mystical substances offered as the explanation or cure for all obscure conditions, but as biologically important substances whose functions place them in the thick of biochemical research. This reviewer was fascinated by the broad application of the antagonism theory in the study of B vitamin metabolism.

A splendid book, in all respects above the general standards of American Chemical Society Monographs. Every biochemist must have access to this work.

WILLIAM A. MOSHER

UNIT OPERATIONS, by George Granger Brown. 611 pages, illustrations, 22 × 29 cm. New York, John Wiley & Sons, Inc., 1950. Price, $7.50.

On the basis that operations common to chemical, metallurgical and other process industries can be better understood if treated individually rather than as part of a particular industry, *Unit Operations* has been prepared by a number of professors of chemical and metallurgical engineering from the Universities of Michigan and Northwestern as a textbook for both graduate and undergraduate students. Unit operations mean such single, complete operations occurring in the various process industries as pulverizing, filtering, evaporating, and electrolyzing.

Following an introduction where unit operations are classified and some practical operations and fundamental concepts are presented, the book is divided into four parts: solids, fluids, separation by mass transfer, and energy and mass transfer rates. The authors have wisely and naturally presented the material in order of increasing difficulty of comprehension. Part I on solids is quite simple and is easily understood. The material covers the elementary properties of solids, their screening reduction and handling for industrial use. Part II on fluids is somewhat more advanced, involving the flow of fluids. It covers briefly the properties of fluids; the flow of solids through fluids; classification, flotation, sedimentation, transportation, pumping and compressing, filtration, centrifugation, and measurement of the flow of fluids; the flow through porous media; and the fluidization of solids. The separation by mass transfer, discussed in Part II, is further advanced. Here solid-liquid and liquid-liquid extraction; vapor-

liquid transfer operations including design and control of fractionating columns, calculation of ideal stages, and distillation and condensation; and adsorption are described. Each chapter in Part III essentially contains a section on equipment, methods of operation, methods of calculation and problems. Part IV on energy and mass transfer rates is the most advanced of all parts, based, as it is, on the elements of heat transfer. Heat transfer, evaporation, crystallization, agitation, mass transfer, and simultaneous heat and mass transfer including psychrometry and drying are discussed.

Generally, the various subjects are treated rather briefly, no doubt due to the complexity of such a large undertaking as the description of process operations. However, the explanation of each operation is complete including equipment used, theory, methods of calculation and design of the equipment, and function. The book contains unusually fine illustrations of the details of such process equipment as crushers, conveyors, meters, pumps, compressors, filters, centrifuges, extractors, heat exchangers, evaporators and crystallizers. Each chapter includes examples and problems illustrating the theory and method of calculation for the design of the various equipment—a great aid to student and engineer alike.

E. W. HAMMER, JR.

HIGH-SPEED COMPUTING DEVICES, by Engineering Research Associates. 451 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1950. Price, $6.50.

This book is better organized and written than any technical book this reviewer ever read. It is an excellent text and reference book on high-speed computing devices.

Computing machines such as desk calculators and punched card business machines are not new. But high-speed electronic computers are new and the great number of components developed for these computers have not been proved by a long test period. The authors avoided getting bogged down in detailed descriptions of particular components but they have integrated a very extensive bibliography into each chapter so that a reader can go to the source material for all the details he wants on any part of the text that strikes his fancy or his needs.

The introduction of each chapter covers the basic considerations for selecting or using a set of components or machines. And in the conclusion, at the end of each chapter, the authors tie up the preceding analysis. There are no loose ends and both the theoretical and the engineering aspects of computing devices are covered.

A chapter on future developments is not based on crystal gazing or wishful thinking; instead, it consists of an analysis of existing electronic components and an effort to show how improvement or adaption might make them useful for a particular function.

The authors have not lost sight of the fact that mathematical problems come in all shapes and sizes. They describe briefly and compare existing computing systems which will handle all sizes of problems. Desk calculators, punched card computers, large scale digital computers and large scale analog computers are all analyzed. This comparison and analysis will be very helpful to people who have a problem and want to know the capabilities of different existing machines.

The book is divided into three parts: basic elements of machine computation, computing systems, and physical components and methods. The first and last parts deal primarily with electronic equipment while the second part describes existing machines of all types. Part one includes a discussion of number theory and numerical analysis as well as a discussion of electronic counters and switches. Part three is a general discussion and comparison of the units which make up large scale electronic computers. Arithmetic elements, storage media, data conversion equipment and special techniques and equipment are included. The book is summed up by a chapter on the factors affecting the choice of equipment.

This book is an excellent reference because of the clear general descriptions of components and systems and the thorough way in which the bibliography is integrated into the subject matter in each chapter. The designer, the user and the student of digital computers will appreciate this text.

E. A. MECHLER

PHOTOGRAPHIC OPTICS, by Allen R. Greenleaf. 214 pages, illustrations, 16 × 24 cm. New York, The Macmillan Co., 1950. Price, $4.75.

Mr. Greenleaf, best known as the originator of the method of cold-light illumination in photographic enlarging now being adopted by most enlarger manufacturers, has written this book with the avowed purpose of removing some of the mythology which seems to have arisen about the attributes of various well-known camera lenses. Heretofore, due to the lack of printed information available, the photographer has been forced for the most part to take the manufacturers' word for the properties of lenses. The author sets out to present in this book an approximate appraisal of different types of lenses without going into the complex mathematical computations involved in complete evaluation.

In his first chapters he explains in concise terms and with a minimum amount of mathematics the general principles of geometrical optics, including a discussion of the effects of the various aberrations and the methods by which they are overcome in actual lenses.

Mr. Greenleaf then proceeds to describe the different types of photographic lenses now used, pointing out their merits and their deficiencies. Patent drawings and specifications for many lenses are included in this section of the book. Methods of testing lenses to evaluate their qualities are also listed.

The final portion of the text is devoted to information on optical accessories. Focussing and shutter mechanisms, supplementary lenses, filters, lens shades, viewfinders and rangefinders are discussed in turn. On the subject of estimating exposure time the author points out several fallacies stemming from the use of various types of exposure meters and guides. The four chapters deal with perspective in photography, projection printing and the obtaining of rectified prints of deformed images, the operational features of slide projectors, and the principles of stereoscopy, respectively.

Useful inclusions at the end of the book are an extensive bibliography and an itemized classification of photographic lens types with a list of many lenses and their makers and the classes to which they are assigned under this system.

The reviewer has only one real criticism with regard to this book. No doubt as a result of his wish to maintain brevity and simplicity, Mr. Greenleaf tends to list the qualities, good and bad, of the many lenses he describes without presenting any theoretical or experimental evidence to back up his statements. Thus the reader is still reduced to taking someone's word—either the manufacturers' or the author's—for the properties of lenses. Some data from test performances or the like might be much more helpful and convincing to a photographer seeking to select a lens for a specific purpose.

In general the book has much to offer in addition to the rating of individual lenses, and it is, therefore, highly recommended for the amateur and the professional photographer who want to know more about the optical principles of photography to guide them in the purchase of new equipment and the more effective use of their present cameras and accessories.

JEAN A. MINKIN

THE INELASTIC BEHAVIOR OF ENGINEERING MATERIALS AND STRUCTURES, by Alfred M. Freudenthal. 587 pages, illustrations, 15 × 22 cm. New York, John Wiley & Sons, Inc., 1950. Price, $7.50.

Practicing engineers sometimes forget that engineering materials are not always isotropic, homogeneous and elastic, and do not always obey the elementary principles of the theory of elasticity. True, without these elementary concepts practical engineering design would be almost impossible; but it is well to remember, particularly in the interpretation of the mechanical behavior of engineering materials from empirical tests, that engineering materials are microscopically heterogeneous and may not be elastic. In *The Inelastic Behavior of Engineering Materials and Structures*, Professor Freudenthal, practicing engineer, educator and research consultant, discusses the various aspects of the inelastic response of structures and material to deformation caused by forces, time and temperature.

Following a very thoughtful introduction concerning the basic concept of matter and methods of approach to the study of the mechanical properties of materials, Professor Freudenthal

has divided his book into three parts: the structural aspect of mechanical behavior, the mechanics of inelastic behavior, and the application of the mechanics of inelastic behavior.

In Part I, the Structural Aspect of Mechanical Behavior, the structure of matter is quite thoroughly explored. The concept of oscillators and quanta as the fundamental form of matter and the new quantum-statistical theory of matter; the forces of interaction in the structure of matter; and the structural theories of deformation including strain hardening, creep and fracture are discussed. In Part II, the Mechanics of Inelastic Deformation, mechanical variables such as forces and displacements; the development of the mechanical equation of state from thermodynamic considerations; elastic and plastic behavior; work hardening, creep and relaxation; and theories of brittle and ductile fracture are presented and discussed. To this point in the volume much of the material included appears infrequently in textbooks. However, in Part III, the Application of the Mechanics of Inelastic Behavior, the material on the design for plasticity and creep is not particularly new. Here is presented, to mention the more significant items, non-uniform bending stress when the plastic range is reached; the thick-walled cylinder under internal and external pressure; such special problems in work-hardening and creep as wire drawing, pressing and rolling of thin metal sheets; the usual material on limitations of elastic design, factor of safety and the theory of limit design; the design for creep of structures and parts at elevated temperatures; and a particularly important chapter—well worth anyone's reading time—on the significance and interpretation of such mechanical tests as those for hardness, tension, torsion, impact, and fatigue.

Probably because of overwriting—a lot of unnecessary words and phrases have been used—the book is not always easy to understand. Although, paradoxically, the text reads easily, the essence of a discussion is frequently buried in a mass of free flowing words and several readings are required before the particular point is unearthed. The use of lengthy phrases and too many adjectives tends to distract rather than enlighten.

E. W. HAMMER, JR.

METALS AT HIGH TEMPERATURES, by Frances H. Clark. 372 pages, illustrations, 16 × 24 cm. New York, Reinhold Publishing Co., 1950. Price, $7.00.

Metals at High Temperature brings together in one volume the most important data on the behavior of metals and alloys at elevated temperatures. The book consists of eleven chapters. The first is principally introductory material, but it also treats the theoretical aspects of creep, stress-strain relations, plasticity and dislocation phenomena. The second chapter, describing the methods and equipment used in making tests at high temperatures, places chief emphasis on the measurement of creep, but also discusses other topics, such as hot fatigue and hardness tests.

In the next five chapters the author presents data on plain carbon steel, chrome iron, moderately and highly alloyed austenitic steels, cobalt-base alloys and nickel-base alloys. One chapter describes the properties of a large number of experimental alloys which were compounded in an effort to find superior materials for high temperature service. Several of these alloys have remarkable properties at high temperatures and thus justify the space devoted to them.

Chapter 9 describes manufacturing processes, including casting, forming, powder metallurgy techniques, and inspection methods. Data on lower melting point alloys are presented in Chapter 10, where the chief emphasis is placed on lead, magnesium, aluminum and the alloys of these metals. The final chapter is devoted to the phenomenon of scaling.

The book not only includes a large amount of tabular data but contains numerous illustrations and graphs. The many photomicrographs shown serve to amplify the material discussed in the text. Throughout the book the author is very thorough in her interpretation of the factual data presented.

This book should prove very valuable to engineers engaged in the design and development of gas turbines, jet aircraft, rockets and other devices in which the high temperature properties of metallic materials are of prime importance.

M. D. EARLE

CHEMICAL THERMODYNAMICS, by Frederick D. Rossini. 514 pages, diagrams, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1950. Price, $6.00.

This is a complete study of chemical thermodynamics based on the experiences of Professor Rossini over a number of years devoted to researches and lectures in various phases of the subject, including the giving of a course in the Graduate School of the National Bureau of Standards. This presentation of the subject presupposes some study of physical chemistry and calculus on the part of the reader. However, an explanation of necessary background material is given in the first five chapters of the book. Included in these chapters is a definition of terms, a survey of the present status of the scale of temperature, and a discussion of fundamental constants, conversion factors, the calorie, and the joule.

The next twenty-five chapters present a comprehensive picture of modern chemical thermodynamics. To simplify instruction from the book, the topics are treated in a logical order rather than in the order of their discovery or development. For example, the chapter on "Thermodynamic Functions from Statistical Calculations" precedes the chapter on "Entropy and the Third Law of Thermodynamics." Although the latter chapter was developed twenty years earlier, an understanding of the statistical calculations greatly facilitates comprehension of the third law and the apparent exceptions to it. It is, therefore, unnecessary for the reader to refer ahead to statements or proof occurring later in the book.

The last five chapters are concerned with the application of the fundamental laws to real physical and chemical systems, thermodynamic calculations, and sources of chemical thermodynamic data. Emphasis is laid throughout the book on the practical application of the laws and principles of thermodynamics. Complete tables of constants and conversion factors contained in the appendix make the use of outside references unnecessary in solving the numerous problems included in the book for testing the reader's understanding of the subject. Also included in the appendix is a table identifying symbols and abbreviations pertinent to the study of chemical thermodynamics. Essentially, the symbols and nomenclature of Lewis and Randall are used. Although exceptions are made to this practice and some additional terms are added, each of these is clearly explained at the point of its introduction.

Historical aspects of the subject, suggested collateral reading, and references to other published works are conveniently grouped at the end of the chapter so as not to complicate presentation of the fundamental concept.

H. EBERT

ELECTROMAGNETIC FIELDS: THEORY AND APPLICATIONS. VOLUME I—MAPPING OF FIELDS, by Ernst Weber. 590 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1950. Price, $10.00.

The subtitle of this book, "Mapping of Fields," concisely describes the nature of its contents. Here is presented a comprehensive survey of the methods of analysis of different means of obtaining electric, magnetic, or other field distributions that can be derived from potential functions. The subject matter is presented primarily by means of numerous illustrative examples which demonstrate the various formulations of basic principles. These are introduced early in the book, to serve as a foundation for the application of field mapping methods which are applicable in hydrodynamical, thermal, and gravitational fields.

Following the chapters on physical principles, the author presents simple applications of the superposition principle, considering such systems as point and line charges, dipole, ring, and rectangular charges and systems of distributed currents. For more complex systems, however, exact analytical solutions are more difficult to obtain. Recourse is made to experimental mapping methods and graphical and numerical field plotting methods. Here again various successful methods are introduced by application to specific problems. The last two chapters of the book, actually about the latter half of the text, are concerned with two and three dimensional analytical solutions, making use of such methods as conjugate functions, conformal mapping, various choices of orthogonal coordinate systems, infinite series expansions and Green's function.

This book is one of the few of its kind which has come to the attention of this reviewer and which so clearly presents the important basic relationships among analytical, experimental, graphical, and numerical methods of solving static field problems. The text is copiously illustrated; the numerous specific references and the large number of problems included at the end of each chapter help make the book an ideal one for graduate courses in this or in allied subjects. It should be, of course, a valuable reference work to electrical engineers and physicists engaged in research development.

S. CHARP

RESPONSE OF PHYSICAL SYSTEMS, by John Dezendorf Trimmer. 268 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1950. Price, $5.00.

This new book on instrumentation and the measurement process is based upon a broad, somewhat philosophical, somewhat analytical, somewhat engineering approach to attempt to describe the output or response of systems which are subject to input or forcing functions. This particular approach to the general problems of instrumentation can of course be applied to such fields of endeavor as sociology, economics and philosophy.

The introductory chapters define the concepts of system response and classify systems and response criteria. This serves as adequate background to treat, rather exhaustively and in some cases with excessive manipulative detail, first and second and higher order systems. These are defined by the order of the differential equation which represents the motion of the system. It is appropriate to mention here that throughout the book the treatment of the linear systems studied is based upon the classical solution of linear differential equations with constant coefficients, with only a reference in the appendix to the applicability of the Laplace transform.

Of particular interest to this reviewer is the chapter devoted to the problem of measuring physical quantities, where the instruments themselves are considered as physical systems, and where the concepts of probability theory are introduced. This subject is becoming increasingly important to those who are concerned with making vast numbers of measurements, especially of varying quantities, and where consideration must be given to the statistical nature of the measured quantities. Treated in lesser detail than the earlier subjects are problems concerned with feedback systems, parametric forcing, distributed systems, and non-linear systems.

This reviewer is of the opinion that this book contributes materially to an understanding of the meaning of system response, but there is doubt as to its direct application as a textbook for a thorough undergraduate course in instrumentation.

S. CHARP

BOOK NOTES

ELECTRONIC ENGINEERING MASTER INDEX, 1949. 296 pages, 17 × 25 cm. New York, Electronics Research Publishing Company, 1950. Price, $17.50.

The present volume is the fourth in a series which has already proved its usefulness. Increased in coverage over the preceding volumes, it indexes nearly 400 magazines, journals, and proceedings. New features have been introduced in this volume, notably the use of volume numbers in the cumulative index to designate the different volumes. Another new feature is the use of an asterisk with articles from some Russian periodicals to indicate an English translation is available at Brookhaven National Laboratory. A work that is of prime importance to the electronics engineer in investigating prior research in the field.

TABLE OF THE BESSEL FUNCTIONS $Y_0(z)$ AND $Y_1(z)$ FOR COMPLEX ARGUMENTS. Prepared by the Computation Laboratory, National Bureau of Standards. 1427 pages, tables, 21 × 27 cm. New York, Columbia University Press, 1950. Price, $7.50.

The present work is another in the useful series of tables which have been produced by the Computation Laboratory. In this the Bessel functions $Y_0(z)$ and $Y_1(z)$ are tabulated for complex arguments defined in polar coordinates along each of the rays $\phi = 0°, 5°, \ldots 90°$, ten

place values of the real and imaginary parts of $Y_0(z)$ and $Y_1(z)$ are given for values of $|z|$ ranging from 0 to 10 at intervals of 0.01. Various auxiliary tables and an explanatory introduction are also included.

IRRIGATION PRINCIPLES AND PRACTICES, by Orson W. Israelson. Second edition, 405 pages, illustrations, diagrams, tables, 15 × 23 cm. New York, John Wiley & Sons, Inc., 1950. Price, $6.00.

Designed primarily as a textbook for college and university students, this work presents fundamental aspects of irrigation principles and practices. It is concerned primarily with the agricultural phases of the subject, although it also considers social and administrative aspects. Particular attention is paid to such topics as water penetration into the soil and the necessity of drainage of irrigated lands. Chapter references, which number over three hundred, have been gathered at the end of the volume.

ELECTRICAL ENGINEERS' HANDBOOK, edited by Harold Pender and Knox McIlwain. Fourth edition, v.p., diagrams and tables, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1950. Price, $8.50.

A companion volume to *Electric Power* noted in the Journal earlier this year, this completes the revision of the *Electrical Engineers' Handbook*. Assisted by a staff of seventy-eight specialists, an increase in the previous edition, the editors have endeavored to reflect the rapid widening of the electronics field. There has been some rearrangement of sections, and the addition of new sections, notably on frequency modulation, pulse techniques and electro-optical devices. In this new edition it continues to be a useful work of reference.

PHYSICAL CHEMISTRY, by Walter J. Moore. 592 pages, illustrations, 15 × 23 cm. New York, Prentice-Hall, Inc., 1950. Price, $5.00.

Prepared by an associate professor of chemistry at the Catholic University, this text book is designed for students in the sciences and in engineering. The author has given derivations of most of the important relationships which presupposes a knowledge of calculus, college physics, and two years of college chemistry. There is included considerable material on atomic and nuclear physics which can well be used for independent reading. The book contains bibliographical apparatus, both as collected references and as footnotes.

CURRENT TOPICS

Deposited Carbon Resistor.—A markedly improved type of deposited carbon resistor, expected to find widespread use in the communications and electronic fields, has been developed at Bell Telephone Laboratories, research and development organization of the Bell System.

In the new resistor, the element boron, as well as carbon, is pyrolitically deposited in a thin film on a suitable ceramic core. The addition of this element, it has been found, gives considerably lower temperature coefficients of resistance than those possessed by plain carbon resistors, as well as good stability. In addition, the new borocarbon resistors provide access to resistance ranges heretofore impossible to attain in stable, accurate film-type resistors. These advantages are expected to permit widespread substitution of borocarbon resistors for the larger and more costly wire wound types.

Announcement of the development of borocarbon resistors was made in a technical paper presented by R. O. Grisdale, A. C. Pfister and G. K. Teal, all of Bell Telephone Laboratories, before the National Electronics Conference in Chicago on September 27.

Recent years have seen an increasing growth in the use of pyrolytic carbon resistors. Known variously as "cracked carbon resistors" or as "high stability carbon film resistors," they are composed of very thin films of microcrystalline carbon formed over the surfaces of ceramic cores. The conducting films of carbon are deposited by the thermal decomposition or pyrolysis of hydrocarbon gases or vapors.

The film type of resistor is particularly useful at high frequencies, for not only can it be made to have small reactance but it is, in effect, all skin so that there is no increase in resistance at high frequencies due to skin effect. The film is also well cooled through its intimate contact with the core and this makes possible the dissipation of large amounts of power per unit area.

While primarily developed for high frequency applications, the pyrolytic carbon resistor possesses other characteristics which have led and are leading to greatly expanded fields of application. Principal among these are the tolerances of one per cent or better attainable in production, the stability in use, the relatively small and predictable temperature coefficient of resistance, and the low noise level. These properties result in large part from the ultimate crystalline structure of the carbon films, which has been closely studied in the Bell Telephone Laboratories.

Pyrolytic carbon films are composed of graphite-like crystallites which are on the average only one ten millionth of an inch in diameter and half as thick and which are so closely and randomly packed together that all resemblance to graphite is lost. Pyrolytic carbon is much harder than carborundum and leaves no smear when rubbed on paper as does graphite itself. Structural and analytical studies have shown however, that each crystallite is surrounded by a skin or layer of complex hydrocarbons which separates it from its neighbors. This hydrocarbon skin increases the specific resistance and the temperature

coefficient of resistance of pyrolytic carbon relative to graphite which, along certain crystal axes, is a good metallic conductor. Decrease in the thickness of this skin or in the total volume it occupies results in a decrease in both the specific resistance and the temperature coefficient of resistance of pyrolytic carbon films.

Unlike thin metal films, the specific resistance of pyrolytic carbon films is independent of film thickness within the limits of experimental error, and films which are on the average only a few hundred millionths of an inch thick have been employed in resistors. The temperature coefficient of resistance for the films, which is always negative, increases from a lower limiting value of about −180 parts per million per degree Centigrade (ppm./°C.) with thick films to about −750 ppm./°C. for very thin films with a resistance per square of about 20,000 ohms. This variation results, in part, from stresses in the film due to the different thermal expansions of the film and its support. On the average, the temperature coefficient of resistance for pyrolytic carbon resistors is about 300 ppm./°C.

Although precision as great as 1 ppm./ °C. in matching the temperature coefficients of resistance of the resistors can be achieved and despite the fact that the statistical variations in temperature coefficient are no greater than for wire wound resistors, the large absolute value of the temperature coefficient of pyrolytic carbon resistors has been a deterrent to their more widespread use. Study has now shown, however, that the addition of boron to the carbon film very greatly decreases the temperature coefficient of resistance and increases its stability as well.

Borocarbon films are produced by the pyrolytic codeposition of boron and carbon from suitable gaseous compounds of these elements. Their temperature coefficients of resistance depend both on film thickness and on boron content. By suitable variation in these, temperature coefficients as small as 20 ppm./° C. can be achieved. Borocarbon resistors of 10 megohm values in half watt sizes with temperature coefficients less than 100 ppm./° C. can be produced, while resistors of 250,000 ohms or lower have temperature coefficients less than 50 ppm./° C. These values of temperature coefficient are entirely comparable with and, indeed, smaller in many cases than for wire wound units of the same resistance value.

The specific resistance of borocarbon films depends on their boron contents, and by suitable variation in composition, films of very high resistances per square can be produced. Values of 500,000 ohms per square are readily obtained and values in excess of 1000 megohms per square have been studied. The temperature coefficients of resistance for these high resistance films are larger than for films of lower resistance, but in the extremely high resistance ranges made accessible through use of these films temperature variations are generally of lesser importance.

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